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VIBRATIONS OF A VISCOELASTIC RIBBED TRUNCATED CONICAL SHELL

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Annotation. In this article, on the basis of the Lagrange variation equation, integral-differential equations of natural vibrations of a viscoelastic ribbed truncated conical shell are obtained. The general research methodology is based on the variational principles of mechanics and variational methods. Geometrically nonlinear mathematical models of deformation of ribbed conical shells are obtained taking into account such factors as discrete introduction of ribs.
Key words: conical shell, panel, nonlinear model, vibrations, viscoelasticity, frequency equation, frequency.

КОЛЕБАНИЯ ВЯЗКОУПРУГОЙ РЕБРИСТОЙ УСЕЧЕННО-КОНИЧЕСКОЙ ОБОЛОЧКИ

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Аннотация. В данной статье на основе вариационного уравнения Лагранжа получена интегро-дифференциальные уравнения собственных колебаний вязкоупругой ребристой усеченной конической оболочки. Общая методология исследования базируется на вариационных принципах механики и вариационных методах. Получены геометрически нелинейные математические модели деформирования ребристых конических оболочек с учетом таких факторов как дискретное введение ребер.
Ключевые слова: коническая оболочка, панель, нелинейная модель, колебания, вязко упругость, частотного уравнения, частота.

VISKOELASTIK QOVURG‘ALI KESILGAN KONUSNING QOBIG‘INING TEBRANISHLARI

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Annotatsiya. Ushbu maqolada Lagranj variatsion tenglamasi asosida qayishqoqelastik qovurg'ali kesik konusli qobig'ining ichki tebranishlarining integral-differentsial tenglamalari olingan. Tadqiqotning umumiy metodologiyasi mexanikaning variatsion printsiplari va variatsion usullarga asoslangan. Qovurg'alarning diskret kiritilishi kabi omillarni hisobga olgan holda qovurg'ali konus membranalarini deformatsiya qilishning geometrik chiziqli bo'lmagan matematik modellari olingan.
Kalit so'zlar: konussimon qobiq, panel, chiziqli bo'lmagan model, tebranishlar, qayishqoqelastik, chastota tenglamalari, chastota.

Introduction.

Conical shell structures are widely used in rocket, aircraft, shipbuilding and construction. To give greater rigidity, the thin-walled part of the shell is reinforced with ribs, while a slight increase in the weight of the structure significantly increases its strength, even if the ribs have a small height.

The study and elimination of resonance phenomena in shells is of great practical interest. A significant number of theoretical and experimental works are devoted to the study of natural vibrations of circular cones. However, there are still no reliable solutions that allow determining the parameters of resonances in a wide range of changes in physical and geometric parameters. There are also works in which the theoretical-experimental method obtained dependences for determining the resonance frequencies [1] and vibration modes of truncated conical panels [2]. Another method is mainly used to study shells, which allow one to go from the equations of stability of conical shells to the corresponding equations for cylindrical shells with a circular cross section. Many works use moment-free and semi-momentless shell theories [3]. Approximate methods are also used to solve problems of natural vibrations [4]. Particularly difficult are the problems of vibrations of reinforced conical shells in a geometrically nonlinear formulation taking into account the rheological properties of the material, solutions for which are practically absent. Analysis of the literature shows that the existing optimal designs of shells for a given geometric and rheological parameters cannot be implemented in practice, the level of research remains only theoretical. In this regard, despite the long history of the solution, the problem of determining the resonant frequency of natural vibrations, taking into account the structural properties of ribbed shells, remains relevant.

The purpose of this work is to develop a methodology, algorithm and program for finding resonant frequencies and waveforms for circular ribbed viscoelastic conical shells under various boundary conditions.

Problem statement and solution methods

Consider a closed circular conical shell with a taper angle θ, thickness h (Fig. 1) with edges *l* and *n* (respectively in the longitudinal and annular directions). To obtain the equations of natural oscillations, we use the principle of possible Lagrange displacements, which takes into account the boundary conditions.

$$\delta(K + \Pi - A) = 0,$$

where *K*- is the kinetic energy of the shell and the rib, Π - is the potential energy of the shell and the rib, and *A*- is the work of external forces.

The middle surface of the shell is taken as the coordinate surface. The X of the Y orthogonal coordinate system directed along the lines of principal curvatures are shown in Fig. 1, and the axis is directed orthogonally to the median surface, towards the concavity. For a conical shell, the Lamé and curvature parameters given in [5] take the following form $A=1, B = x\sin\theta$, $k_x = 0,\ k_y = \frac{\text{ctg}\,\theta}{x}$.

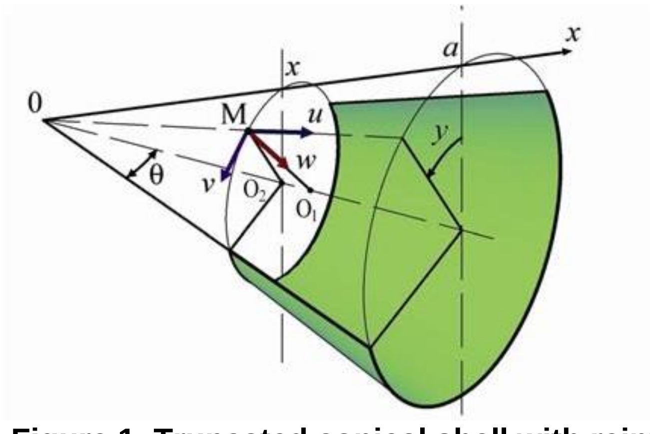

Figure 1. Truncated conical shell with reinforced ribs

Deformations in the shell coordinate surface are expressed through displacements along the $u,\ v,\ w$ axes, $x,\ y,\ z$ respectively, as follows

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}+\frac{1}{2}\cdot\left(\frac{\partial w}{\partial x}\right)^2;$$ (1)

$$\varepsilon_{yy}=\frac{1}{x\sin\theta}\cdot\frac{\partial v}{\partial y}+\frac{u}{x}-\frac{\mathrm{ctg}\,\theta}{x}w+\frac{1}{2}\cdot\left(\frac{1}{x\sin\theta}\cdot\frac{\partial w}{\partial y}+\frac{\mathrm{ctg}\,\theta}{x}v\right)^2;$$

$$\gamma_{xy}=\frac{\partial v}{\partial x}+\frac{1}{x\sin\theta}\cdot\frac{\partial u}{\partial y}-\frac{v}{x}+\frac{\partial w}{\partial x}\cdot\left(\frac{1}{x\sin\theta}\cdot\frac{\partial v}{\partial y}+\frac{\mathrm{ctg}\,\theta}{x}v\right).$$

Deformations in a layer at a distance from z the coordinate surface, taking into account the transverse shears, have the form

$$(u^z=u+z\cdot\psi_x,\ v^z=v+z\psi_y,\ w^z=w)$$

$$\varepsilon_x^z=\varepsilon_{xx}+z\cdot\chi_1\quad;\quad\varepsilon_y^z=\varepsilon_{yy}+z\cdot\chi_2\quad;\quad\gamma_{xy}^z=\gamma_{xy}+2\cdot z\cdot\chi_{12}$$ (2)

and besides

$$\gamma_{xz}=kg(z)\cdot\left(\psi_x+\frac{\partial w}{\partial x}\right);\quad\gamma_{yz}=k\,g(z)\cdot\left(\psi_y+\frac{1}{x\sin\theta}\cdot\frac{\partial w}{\partial y}+ctg\,\theta\frac{v}{x}\right)$$ (3)

Here $\psi_x,\ \psi_y$- the angles of rotation of the normal segment at the coordinate surface in sections XOZ and YOZ, respectively; $g(z)$- function that describes the distribution τ_{xz} and τ_{yz} shear stresses; k =const.

Functions characterizing changes in curvature $\chi_1,\ \chi_2$and torsion χ_{12}have the form [6]

$$\chi_1=\frac{\partial\psi_x}{\partial x};\ \chi_2=\frac{1}{x\sin\theta}\cdot\frac{\partial\psi_y}{\partial y}+\frac{\psi_x}{x};\ 2\chi_{12}=\frac{\partial\psi_y}{\partial x}+\frac{1}{x\sin\theta}\cdot\frac{\partial\psi_x}{\partial y}-\frac{\psi_y}{x}.$$

Physical relations for an isotropic viscoelastic body take the form [7]

$$\sigma_x=\frac{\tilde{E}}{1-\nu^2}(\varepsilon_x^z+\nu\varepsilon_y^z);\ \sigma_y=\frac{\tilde{E}}{1-\nu^2}\cdot(\varepsilon_y^z+\nu\varepsilon_x^z);\ \tau_{xy}=\frac{\tilde{E}}{2(1+\nu)}\gamma_{xy}^z;\tau_{xz}=\frac{\tilde{E}}{2(1+\nu)}\gamma_{xz}\ ;\tau_{yz}=\frac{\tilde{E}}{2(1+\nu)}\gamma_{yz}.$$

Here μ- the Poisson's ratio of the shell material, which is considered constant; $\tilde{E}_\kappa$ - operator moduli of elasticity of the conical shell and rib

$$\tilde{E}_k[f(t)]=E_{0\kappa}\left[f(t)-\int_0^t R_{E\kappa}(t-\tau)f(\tau)d\tau\right],$$ (4)

$E_{0\kappa}$ - Young's instant modulus of elasticity (k = 1, 2, 3 ... L); к=1- instantaneous modulus of elasticity of the shell, к=2,3... L-instant modulus of elasticity of the ribs, $f(t)$ - is a continuous function; $R_{E\kappa}(t-\tau)$ - the core of relaxation.

The physical relations, taking into account the creep of the material (5) on the basis of the linear theory of heredity, take the form [7]

$$\sigma_x=\frac{E_0}{1-\nu^2}\left[\varepsilon_x^z+\nu_1\varepsilon_y^z-\int_0^t(\varepsilon_x^z+\nu_1\varepsilon_y^z)\,R_{E1}(t-\tau)d\tau\right];\ \sigma_y=\frac{E_0}{1-\nu^2}\left[\varepsilon_y^z+\nu_1\varepsilon_x^z-\int_0^t(\varepsilon_y^z+\nu_1\varepsilon_x^z)\,R_{E1}(t-\tau)d\tau\right];$$

$$\tau_{xy}=\frac{E_0}{2(1+\nu)}\left[\gamma_{xy}^z-\int_0^t\gamma_{xy}^z\,R_{E2}(t-\tau)d\tau\right];\ \tau_{zx}=\frac{E_0}{2(1+\nu)}\left[\gamma_{zx}^z-\int_0^t\gamma_{zx}^z\,R_{E2}(t-\tau)d\tau\right],\tau_{yz}=\frac{E_0}{2(1+\nu)}\left[\gamma_{yz}^z-\int_0^t\gamma_{yz}^z\,R_{E2}(t-\tau)d\tau\right]$$ (5)

Here $R_{E1}(t-\tau), R_{E2}(t-\tau)$- are the cores of relaxation. The influence of rib stiffness is taken into account using the Dirac impulse function.

The location and height of the ribs is set by the function

$$H(x,y) = \sum_{j=1}^{m} h^j \bar{\delta}\ (x-x_j) + \sum_{i=1}^{n} h^i \bar{\delta}(y-y_i) - \sum_{i=1}^{n} \sum_{j=1}^{m} h^{i\ j} \bar{\delta}\ (x-x_j)\ \bar{\delta}\ (y-y_i) \tag{6}$$

Integrating voltages (4) by z, in the range from $-\frac{h}{2}$ to $\frac{h}{2}+H$, we obtain the forces, moments and shear forces reduced to the middle surface of the shell, for a unit length of the middle surface

$$N_x = \tilde{G}_1[(h+\bar{F})\cdot\varepsilon_1 + \bar{S}\psi_1]\ ;\ N_y = \tilde{G}_2[(h+\bar{F})\cdot\varepsilon_2 + \bar{S}\psi_2]\ ;\ N_{xy} = \tilde{G}_{12}[(h+\bar{F})\gamma_{xy} + \bar{S}\psi_{12}];$$

$$M_x = \tilde{G}_1\left[\bar{S}\varepsilon_1 + \left(\frac{h^3}{12}+\bar{J}\right)\psi_1\right]\ ;\ M_y = \tilde{G}_2\left[\bar{S}\varepsilon_2 + \left(\frac{h^3}{12}+\bar{J}\right)\cdot\psi_2\right], \tag{7}$$

$$M_{xy} = \tilde{G}_{12}\left[\bar{S}\gamma_{xy} + \left(\frac{h^3}{12}+\bar{J}\right)\psi_{12}\right];\ Q_x = k\tilde{G}_{13}(h+\bar{F})\cdot\left(\psi_x + \frac{\partial W}{\partial x}\right),\ \ Q_y = k\tilde{G}_{23}(h+\bar{F})\left(\psi_y + \frac{1}{x\sin\theta}\frac{\partial W}{\partial y} + \frac{ctg\,\theta}{x}V\right)$$

where

$$\varepsilon_1 = \varepsilon_{xx} + \nu\varepsilon_{yy},\ \varepsilon_2 = \varepsilon_{yy} + \nu\varepsilon_{xx},\ \psi_1 = \chi_1 + \nu\chi_2,\ \psi_2 = \chi_2 + \nu\chi_1, \psi_{12} = 2\chi_{12},$$

$$\tilde{G}_1[f(t)] = \tilde{G}_2[f(t)] = \frac{\tilde{E}}{1-\nu^2}[f(t)] = \frac{E_0}{1-\nu^2}\left(f(t) - \int_0^t R_E(t-\tau)f(\tau)d\tau\right),$$

$$\tilde{G}_{12}[f(t)] = \tilde{G}_{13}[f(t)] = \tilde{G}_{23}[f(t)] = \frac{\tilde{E}[f(t)]}{2(1+\nu)} = \frac{E_0}{2(1+\nu)}\left(f(t) - \int_0^t R_E(t-\tau)f(\tau)d\tau\right).$$

$\bar{F}, \bar{S}, \bar{J}$ - are the areas (transverse or longitudinal) of the ribs section per unit length of the median surface. The static moment and moment of inertia of the middle surface of the shell have the form

$$\bar{F} = \int_{h/2}^{h/2+H} dz\ \ ; S = \int_{h/2}^{h/2+H} zdz;\ \bar{J} = \int_{h/2}^{h/2+H} z^2 dz.$$

Let a transverse dynamic load act on the shell $q(x,\ y,\ t)$. Then the unknown desired displacement functions $U,\ V,\ W$ and the angles of rotation of the normal $\psi_x,\ \psi_y$ are functions of the following variables x, y and t.

The functional of the total energy of deformation of the viscoelastic shell has the form

$$J = \int_{t_0}^{t_1}(K - \Pi + A)dt \tag{8}$$

Here K - kinetic energy of the system, Π — potential energy of the system, A - the work of external forces, were

$$K = \frac{\rho}{2}\int_{a_1}^{a}\int_0^b\int_{-\frac{h}{2}}^{\frac{h}{2}+H}[(\dot{u}^z)^2 + (\dot{v}^z)^2 + (\dot{w}^z)^2]\,x\sin\theta\,dxdy\,dz = \frac{\rho}{2}\int_{a_1}^{a}\int_0^b[(h+\bar{F})(\dot{u}^2+\dot{v}^2+\dot{w}^2) + 2\bar{S}(\dot{u}\dot{\psi}_x + \dot{v}\dot{\psi}_y) + \left(\frac{h^3}{12}+\bar{J}\right)(\dot{\psi}_x^2 + \dot{\psi}_y^2)]\,x\sin\theta\,dxdy \tag{9}$$

$$Э = \Pi - A = \frac{1}{2}\int_{a_1}^{a}\int_o^b\left[N_x\varepsilon_{xx} + N_y\varepsilon_{yy} + N_{xy}\gamma_{xy} + M_x\chi_1 + M_y\chi_2 + 2M_{xy}\chi_{12} + Q_x\left(\psi_x + \frac{\partial w}{\partial x}\right) + Q_y\left(\psi_y + \frac{1}{x\sin\theta}\cdot\frac{\partial w}{\partial y} + \frac{ctg\,\theta}{x}v\right) - 2qw\right]x\sin\theta\,dxdy \tag{10}$$

In the above formulas ρ - shell material density; in expressions (9) and (10), the dots denote the derivatives of the variable t; $b = 2\pi$ - for a conical shell.

The energy can be expressed in terms of deformations, then the expression (10) is represented as follows

$$Э=\frac{\tilde{E}}{2(1-\mu^2)}\int_{a_1}^{a}\int_0^b\Big[(h+\overline{F})\Big(\varepsilon_{xx}^2+\varepsilon_{yy}^2+2\mu\varepsilon_{xx}\varepsilon_{yy}+\mu_1\gamma_{xy}^2+\frac{5}{6}\mu_1\left(\psi_x+\frac{\partial w}{\partial x}\right)^2+\frac{5}{6}\mu_1\left(\psi_y+\frac{1}{x\sin\theta}\frac{\partial w}{\partial y}+\frac{ctg\,\theta}{x}v\right)^2\Big)+2\bar{S}(\chi_1\varepsilon_{xx}+\chi_2\varepsilon_{yy}+\mu\chi_2\varepsilon_{xx}+\mu\chi_1\varepsilon_{yy}+2\mu_1\gamma_{xy}\chi_{12})+\left(\frac{h^3}{12}+\bar{J}\right)(\chi_1^2+\chi_2^2+2\mu\chi_1\chi_2+4\mu_1\chi_{12}^2)-2(1-\mu^2)\frac{q}{E}w\Big]x\sin\theta\,dxdy \quad (11)$$

where $\mu_1=\frac{1-\nu}{2}$. If the conical viscoelastic shell is closed, then $a_1=0$.

Consider a reinforced conical shell with narrow edges. The problem under consideration is solved in dimensionless parameters. Then the basic relationship takes the following form

$$\xi=\frac{x}{a},\quad \eta=\frac{y}{b},\quad \lambda=\frac{a}{bx\sin\theta},\quad \lambda=\frac{\lambda_1}{\xi},\quad \bar{U}=\frac{au}{h^2},$$
$$\bar{V}=\frac{bx\sin\theta\, v}{h^2},\quad \bar{W}=\frac{w}{h},\bar{\psi}_x=\frac{a\psi_x}{h},\bar{\psi}_y=\frac{bx\sin\theta\psi_y}{h},$$
$$\bar{a}=\frac{a}{h},\quad \bar{P}=\frac{a^4P}{Eh^4},\quad \bar{t}=\frac{h}{a^2}\sqrt{\frac{E_0}{(1-\nu^2)\rho}}t,\quad \overline{F}=\frac{\bar{F}}{h},\bar{J}=\frac{\bar{S}}{h^2},\quad \bar{J}=\frac{\bar{J}}{h^3} \quad (12)$$

then we get the following expressions for kinetic and potential energies

$$\bar{K}=\frac{1}{\bar{a}^2}\int_{a_1}^{1}\int_0^1\left[\left(1+\bar{\bar{F}}\right)\left(\dot{\bar{U}}^2+\lambda^2\dot{\bar{V}}^2+\bar{a}^2\dot{\bar{W}}^2\right)+2\bar{\bar{S}}\left(\dot{\bar{U}}\dot{\bar{\psi}}_x+\lambda^2\dot{\bar{V}}\dot{\bar{\psi}}_y\right)+\left(\frac{1}{12}+\bar{J}\right)\left(\dot{\bar{\psi}}_x^2+\lambda^2\dot{\bar{\psi}}_y^2\right)\right]\xi d\xi d\eta \quad (13)$$

$$\bar{Э}=\int_{\bar{a}_1}^{1}\int_0^1\Big[\left(1+\bar{\bar{F}}\right)\Big(\bar{\varepsilon}^2+\lambda^4\bar{\varepsilon}_{yy}^2+2\nu\lambda^2\bar{\varepsilon}_{xx}\bar{\varepsilon}_{yy}+\mu_1\lambda^2\bar{\gamma}_{xy}^2+\frac{5}{6}\mu_1\bar{a}^2\cdot\left(\bar{\psi}_x+\frac{\partial\bar{W}}{\partial\xi}\right)^2+\frac{5}{6}\mu_1\lambda^2\bar{a}^2\cdot\left(\bar{\psi}_y+\frac{\partial\bar{W}}{\partial\eta}+\frac{c_3}{\xi}\bar{V}\right)^2\Big)+2\bar{\bar{S}}(\bar{\chi}_1\bar{\varepsilon}_{xx}+\lambda^4\bar{\chi}_2\bar{\varepsilon}_{yy}+\nu\lambda^2\bar{\chi}_2\bar{\varepsilon}_{xx}+\nu\lambda^2\bar{\chi}_1\bar{\varepsilon}_{yy}+2\mu_1\lambda^2\bar{\gamma}_{xy}\bar{\chi}_{12})+\left(\frac{1}{12}+\bar{J}\right)\cdot(\bar{\chi}_1^2+\lambda^4\bar{\chi}_2^2+2\nu\lambda^2\bar{\chi}_1\bar{\chi}_2+4\mu_1\lambda^2\bar{\chi}_{12}^2)-2(1-\nu^2)\bar{P}\bar{W}\Big]\xi d\xi d\eta \quad (14)$$

The variational equation, for a thin viscoelastic shell supported by edges of the *l* - and *j* - directions, is obtained as a variation from the sum of potential and kinetic energies, taking into account the conjugation condition [8, 9]

$$\delta K+\sum_{l=1}^{N}\delta K_l+\sum_{j=1}^{n}\delta K_l+\delta Э+\sum_{l=1}^{m}\delta Э_l=0, N=n+m \quad (15)$$

where $Э_l=П_l-A_l$- the potential difference of external forces and work applied to the edges. This sum contains as many terms of equations (15) as there are edges in the corresponding directions. Thus, in relation to the considered ribbed shell, the Lagrange principle can be formulated as follows [10]: actual displacements of the median surface of the shell $u,\ v,\ w$ and edges $u_m,\ v_m,\ w_m(m=k,j)$, corresponding to these boundary conditions and the load, and transferring the shell from its natural position to a state of elastic equilibrium, differ from all possible displacements in that they inform the system in question of the minimum amount of potential energy.

The proper oscillations of a conical shell with freely supported at the ends are considered.

Thus, the mathematical formulation of the problem is formulated and the basic relations of viscoelastic conical shells with their own vibrations are given, which take into account geometric nonlinearity, discrete introduction of viscoelastic ribs, their shear and torsional stiffness, transverse shifts and inertia of rotation.

Solution methods.

It is assumed that the integral terms in (4)–(6) are small. Then for the function $f(t)$ there is a function $f(t) = \phi(t)e^{-i\omega_R t}$ and integral terms are replaced by the following expressions[16]

$$\bar{E}_\kappa \phi = E_{0\kappa}[1 - \Gamma_\kappa^C(\omega_R) - i\Gamma_\kappa^S(\omega_R)]\phi,$$

where $\Gamma^C(\omega_R) = \int_0^\infty R_E(\tau)\cos\omega_R\tau\, d\tau,\ \Gamma^S(\omega_R) = \int_0^\infty R_E(\tau)\sin\omega_R\tau\, d\tau$

responsible for the cosine and sine of the Fourier transform, ω_R - the actual value. The calculations use a three - parameter Koltunov - Rzhanitsyn kernel

$$R_k(t) = A_k e^{-\beta_k t}/t^{1-\alpha_k}$$

To calculate the dynamic characteristics of the truncated conical shells, taking into account the geometric nonlinear terms (1) - (4), we neglect and use the finite element method (FEM) in displacements. Consider 8-node isometric curved finite elements (FE), the so-called "degenerate" shell element [11]. The element is designed to calculate shells of medium and small thickness with ribs. The geometry of the FE represents a curved parallelepiped in three-dimensional space with a linear surface in thickness. Used local ξ, η, ζ and global cartesian *x, y, z* coordinate systems. The coordinates of an arbitrary point of the FE are expressed in terms of the coordinates of the nodal points $\bar{r}_i$ and components of the unit normal vector $\bar{n}_i$. Finite element representation of the equilibrium equations of a finite element system in a state of motion, taking into account (5), (6), (9) and (15) has the form [12].

$$[M]\{\ddot{q}\} + [\bar{K}(\omega_R)]\{q\} + \{R\} = 0, \quad (16)$$

where $[M] = \sum_{i,j=1}^{n}[m_{ij}]$- matrix mass of the system ($[m_{ij}] = \alpha_1[m_{ij}]_a + \beta_1[m_{ij}]_p$, $[m_{ij}]_a$- elements matrix of masses of a truncated conical shell, $[m_{ij}]_p$- elements matrix of the masses of the reinforcing rod, (α_1, β_1-dimensionless coefficients), $\{R\}$- vector of external loads, $[\bar{K}(\omega_R)]$- system stiffness matrix ($[K] = \sum_{i,j=1}^{n}[k_{ij}]$-stiffness matrix of the conical shell panel and the reinforcing rod), $\{q\}$- unknown nodal movements, $\{R\}$- vector of external loads. The mass matrix (16) is consistent: structure of the mass matrix $[M] = \sum_{i,j=1}^{n}[m_{ij}]$ completely coincides with the structure of the stiffness matrix. Both matrices ($[M]$,$[\bar{K}(q^t, \omega_R)]$) have size (NxN), which corresponds to the number of degrees of freedom of the FE.

It is assumed that $\{R\}$=0, then the proper oscillations of the truncated conical shell are considered. The solution (16) is sought in the form

$$\{q\} = \{Q_A\}e^{-i\omega t} \quad (17)$$

where $\{Q_A\}$- amplitudes of unknown nodal displacements, complex value; $\omega = \omega_R + i\omega_I$ - the complex frequency to be determined.

Substituting (17) into (16), we obtain the following homogeneous algebraic equation

$$(-\omega^2[M] + [K(\omega_R, q^\circ)])\{Q_A\} = 0 \quad (18)$$

The complex roots of the frequency equation (18) are determined by the Muller method, at each iteration of the Muller method, the Gauss method is applied with the allocation of the main element [13].

Numerical results and analysis

The radii (large and small), the height and thickness of the shell, the angle of the semi-solution of the truncated cone, the modulus of elasticity, the Poisson's ratio, the parameters of the relaxation core of the material and the geometric and mechanical parameters of the edges should be set as initial data. As the relaxation core of a viscoelastic material, we take a three-parameter core $R(t) = \frac{Ae^{-\beta t}}{t^{1-\alpha}}$ Rzhanitsyn–Koltunova [14], which has a weak singularity. Here A, α, β - parameters of materials.

Table 1. Frequency change depending on the thickness of the shell

n	*h*	$\omega = \omega_R + i\omega_I$	
		ω_R	$-\omega_I\,10^{-2}$
1	0.05	1.94894	1.77381
2		2.18942	2.57321
3		3.44160	2.90566
4		3.92574	3.04574
1	0.02	1.31147	1.42621
2		1.53439	2.49433
3		3.54578	2.98778
4		4.12429	3.21429
1	0.01	1.05218	1.22718
2		1.26867	1.36860
3		3.55691	3.45697
4		4.53974	3.65924

Viscoelastic truncated conical shells are considered, in which the large bases are pivotally supported, and the second bases are freely supported on the conical shell. A conical shell supported by 4 ribs, 4*h* high and 2*h* wide (the lengths of the arcs between the ribs are the same- $s_i = \pi a / 2$).

All ribs are viscoelastic with the same rheological properties. The parameters of a truncated conical shell made of plexiglass have the following values: taper angle θ =0.20, 0.40, 0.60, 0.80, radii of the base of the truncated conical shell - $a_1 = 15м$, $a = 18м$ (the length of the shell is 10 meters). The physical and mechanical characteristics of the rib and shell, respectively, take the following values: $\rho_c = 7{,}8 \cdot 10^3 \frac{кг}{м^3}, \rho_0 = 3 * 10^3 \frac{кг}{м^3}, \nu_c = 0.25, \nu_0 = 0.35$, E_c= 2x10^{12} gPa, E_0=20x10^{12} Pa. The values of rheological parameters are taken as: $A = 0{,}048; \beta = 0{,}05; \alpha = 0{,}1$

Table 1 shows the complex values of the lower frequencies of a dissipatively homogeneous reinforced (with four rods) truncated conical shell at different shell thicknesses in the limit of the Kirchhoff-Love hypothesis. The values of the complex natural frequency and the corresponding forms of oscillations are determined when both ends of the shell contour are pivotally supported ($N_1 = M_1 = U = V = W = 0$). In the case under consideration, axisymmetric oscillations correspond to the minimum natural frequencies (real parts of complex frequencies). Analysis of the calculation results allows us to conclude that with a decrease in the thickness of the viscoelastic conical shell, in its real and imaginary parts, the first and second oscillation frequencies are monotonically killed. The real parts of the third and fourth frequencies decrease moderately, and the corresponding imaginary parts gradually increase. The results of calculations of homogeneous viscoelastic mechanical systems are shown in Fig.2 and 3.

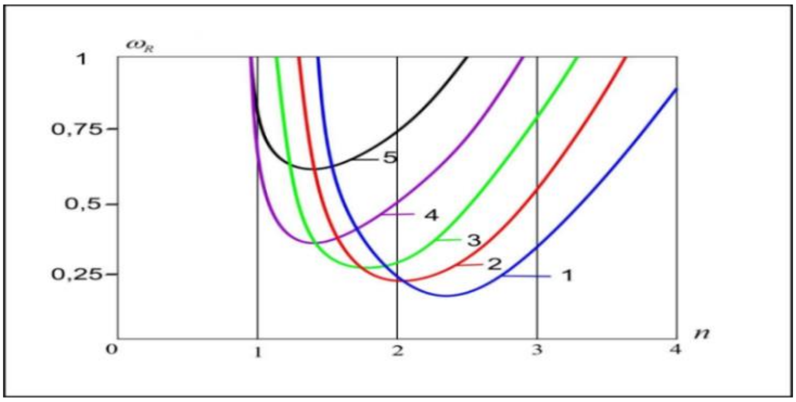

Fig. 2. Change of the parameter (real part) of the complex frequency of truncated conical shells of rotation depending on n at various γ **: 1.** θ**=0; 2.** θ**=0.20;3.** θ**=0.40; 4.** θ**=0.60; 5.** θ**=0.80.**

As can be seen from Fig. 2 and Fig. 3, reinforcement of the shell with four longitudinal ribs makes it possible to increase the real and imaginary parts of the natural frequency of the truncated conical shell. Parameter γ=0- corresponds to a reinforced cylindrical shell, θ=0.20, 0.40, 0.60, 0.80 – correspond to the taper angle of the reinforced truncated conical shell.

Taking into account the rheological properties of the material allow you to increase (or decrease) the frequency values of the shell up to 10%. For bending vibrations, there is a significant decrease in the local maxima of normal displacements with an increase in the area of the annular edges of the shell.

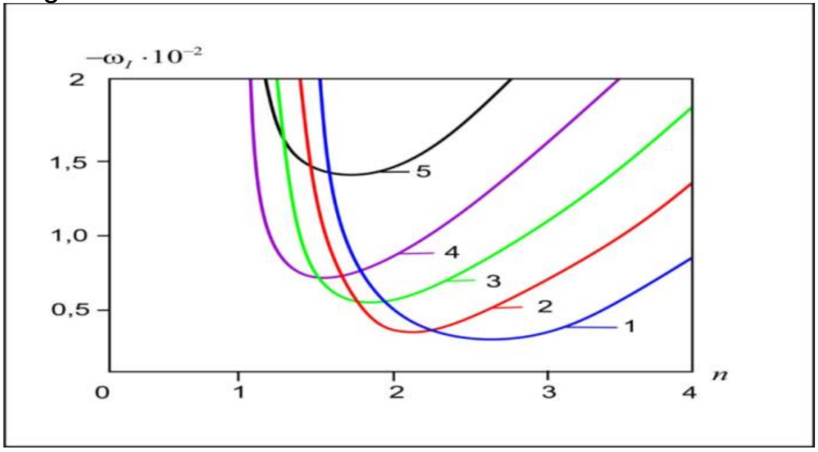

Fig. 3. Change of the parameter (imaginary part) of the complex frequency of the truncated conical shell depending on n at various γ ***:***

θ=0; 2. θ=0.20;3. θ=0.40; 4. θ=0.60; 5. θ=0.80.

This effect becomes more noticeable with an increase in the frequency number n.

Conclusion.

1. Algorithms for solving the problems of natural vibrations of shells for ribbed viscoelastic conical shells have been developed. The finite element method, the freezing method, and the Muller and Gauss methods are used to solve dynamic problems;

2. Analysis of the calculation results shows that with a decrease in the thickness of the viscoelastic conical shell, the real and imaginary parts of the first and second oscillation frequencies monotonically decrease. The real parts of the third and fourth frequencies decrease moderately, and the corresponding imaginary parts gradually increase;

3. Taking into account the geological properties of the material allow you to increase (or decrease) the frequency values of the shell up to 10%.

References

[1]. Новожилов В. В. Теория тонких оболочек. – Л.: Судпромиздат, 1962.- 431с.

[2]. Вольмир А.С. Нелинейная динамика пластинок и оболочек. – М.: Наука, 1972. 432 с.

[3]. Safarov I. I, Almuratov, Ш. TeshaevM.Kh, Homidov F.F., Rayimov D.G. On the dynamic stress-strain state of isotropic rectangular plates on an elastic base under vibration loads//Indian Journal of Engineering.17(47), 2020. PP. 127-133

[4]. Teshaev, M.K., Safarov, I.I., Kuldashov, N.U., Ishmamatov, M.R., Ruziev, T.R. On the Distribution of Free Waves on the Surface of a Viscoelastic Cylindrical Cavity // Journal of Vibrational Engineering and Technologies.

[5]. Кованич С.Ф. Метод конечных элементов в нелинейных задачах инженерной механики. «Запорожье» Р.Украина ,2009.400с.

[6]. Safarov I.I., Teshaev M.Kh., Akhmedov M.S. Free Oscillations of a Toroidal Viscoelastic Shell with a Flowing Liquid. American Journal of Mechanics and Applications. 2018; 6(2): - Pp 37-49 http://www.sciencepublidoi:10.11648

[7]. Абовский Н.П. Ребристые оболочки. Красноярск. 1967. 61с.

[8]. Ершов, Н. П. Состояние и перспективы развития расчетно-экспериментальных работ в области проектирования тонкостенных конструкций из композиционных материалов// Механика композитных материалов. – 1988.– № 1.– С. 86-92.

[9]. Ishmamatov MR. Urinov Sh. R., Yuldoshev Sh.Y., Tursinboeva Z.U. Effect of initial concentration distribution comfortable on the facade parameters. InInvestigations of fluid movement filtration of solutions in elastic regime of the plaster//Proceedings of the international conference on integrated innovative development of Zarafshon region: achievements, challenges and prospect 2017 (pp. 193-197).

[10]. Ефимов А.Б., Аксененко О. В., Цвелих А. В. Решение осесимметричной задачи теории упругости для несжимаемых материалов с помощью гибридного метода конечных элементов // Математическое моделирование систем и процессов. – 1(1). – 1992. – С. 67-81

[11]. Maiboroda, V.P., Troyanovskii, I.E., Safarov, I.I., Vazagashvili, M.G., Katalymova, I.V. Wave attenuation in an elastic medium. Journal of Soviet Mathematics, 1992, 60(2), стр. 1379–1382

[12]. Sobotka, Z. Free vibration of visco-elastic orthotropic rectangular plates// Acta Technica CSAV, 6, 678–705, (1978).

[13]. Kaplunov J.D., Wilde M.V. Edge and interfacial vibrations in elastic shells of revolution // J. Appl. Math. Phys. (ZAMP). 2000. - 51. - P. 29-48.

[14]. Safarov I.I., Teshaev M.Kh., Boltayev Z.I. Propagation of linear waves in multilayered structural - Inhomogeneous cylindrical shells// Journal of Critical Reviews, 2020, 7(12), p. 893-904 http://jcreview.com/