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BUILDING THE STATE SPACE REPRESENTATION OF RLC CIRCUIT

Keldiyarova Malika Shukhrat qizi – Karshi engineering economics institute, assistant teacher of "Automation of technological processes and controlling" department

Annotation. The main target of the article is to analyze the state space representation of an electrical system. During the research many useful information about the mathematical modelling of linear systems in state space representation and linear and time invariant (LTI) systems were gathered.
Keywords: State space representation, linear and time invariant, state, capacitor, inductor, voltage, current, resistor, state variable, single input multiple output systems, voltage source.

ПОСТРОЕНИЕ ПРОСТРАНСТВО СОСТАЯНИЙ ПРЕДСТАВЛЕНИЯ ЭЛЕКТРИЧЕСКОЙ ЦЕПИ RLC

Келдиярова Малика Шухратовна – Каршинский инженерно-экономический институт, ассистент кафедры «Автоматизация технологических процессов и контроль».

Аннотация. Основная цель статьи - проанализировать представление электрической системы в пространстве состояний. В ходе исследования было собрано много полезной информации о математическом моделировании линейных систем в представлении в пространстве состояний и линейных и инвариантных во времени систем.
Ключевые слова: Представление в пространстве состояний, линейное и неизменное во времени, состояние, конденсатор, катушка индуктивности, напряжение, ток, резистор, переменная состояния, системы с одним входом и несколькими выходами, источник напряжения.

RLC ELEKTR ZANJIRINING FAZODAGI HOLAT TENGLAMASINI QURISH

Keldiyarova Malika Shuxratovna - Qarshi muhandislik-iqtisodiyot instituti, Texnologik jarayonlarni avtomatlashtirish va boshqarish kafedrasi assistenti.

Annotatsiya. Maqolaning asosiy maqsadi - elektr zanjirining fazodagi holat tenglamasini tahlil qilish. Tadqiqot davomida chiziqli tizimlarni fazoda o'zgarmas vaqt davomiyligida modellashtirisha to'g'risida foydali ma'lumotkar berilgan
Kalit so'zlar: Holat fazosida aks etish, vaqt bo'yicha chiziqli va doimiy, holat, kondensator, induktor, kuchlanish, oqim, qarshilik, holat o'zgaruvchisi, bitta kirish va ko'p chiqishga ega tizimlar, kuchlanish manbai.

The state-space description provide the dynamics as a set of coupled first-order differential equations in a set of internal variables known as *state variables*, together with a set of algebraic equations that combine the state variables into physical output variables.

A standard form for the state equations is used throughout system dynamics. In the standard form the mathematical description of the system is expressed as a set of n coupled first-order ordinary differential equations, known as the *state equations*, in which the time derivative of each state variable is expressed in terms of the state variables

A system *output* is defined to be any system variable of interest. A description of a physical system in terms of a set of state variables does not necessarily include all of the variables of direct engineering interest.

The complete system model for a linear time-invariant system consists of a set of n state equations, defined in terms of the matrices **A** and **B**, and a set of output equations that relate any output variables of interest to the state variables and inputs, and expressed in terms of the **C** and **D** matrices. The task of modeling the system is to derive the elements of the matrices, and to write the system model in the form:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

With constant matrices A, B, C, D is always possible. This representation is the most common for systems that are linear and time invariant.

The procedure to build the state space representation of a system is analogous to the decomposition of a higher order linear differential equation to a system of first order differential equations, once the states, the input and the outputs are defined. The first thing to do is to fix the states of the system, that are those quantities that are related to the others by means of an integral relation and thus take into account the memory of the system. Once the states are defined, their variation must be expressed in function of the input and of the states themselves. Consider the following RLC circuit:

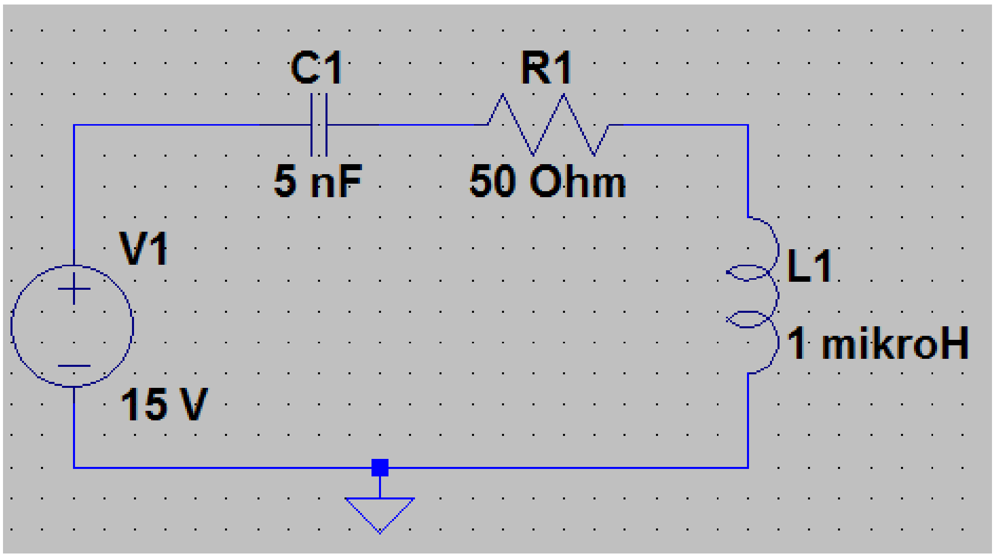

Fig.1. RLC circuit.

The two components that adds a state to the system are the inductor and the capacitor, for which the following relations must be satisfied:

$$v_L = L\frac{di_L}{dt}$$

$$v_C = C\frac{dv_C}{dt}$$

From these, we understand that the two states of the system are the voltage across the capacitor and the current through the inductor, since their instantaneous value is find through the integration of the other electrical variable:

$$X = \begin{pmatrix} i_L \\ v_C \end{pmatrix}$$

Where the time dependency is implied. The input of the system in this case is the voltage source V_G. We must express the variation of these quantities as a linear combination of themselves and of the inputs. To begin we can invert the relations:

$$\frac{di_L}{dt} = \frac{v_L}{L}$$

$$\frac{dv_C}{dt} = \frac{i_C}{C}$$

Now, the voltage across the inductor and the current through the capacitor must be expressed in terms of the states and the input. From the Kirchhoff voltage law we can state:

$$v_L = V_g - v_c - i_L R$$

For the second equation, the things are easier since the current through the capacitor is the same as the current through the inductor, a state variable; this leads us to:

$$\frac{di_L}{dt} = \frac{V_g - v_c - i_L R}{L}$$

$$\begin{cases} \frac{di_L}{dt} = \frac{V_G - v_C - i_L R}{L} \\ \frac{dv_C}{dt} = \frac{i_L}{C} \end{cases}$$

From the system above we must, define the A, B, C, D matrices by inspection; it is easy to find out that:

$$A=\begin{pmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{pmatrix}, \quad B=\begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix}$$

The above system can now be written by means of the vector equation:

$$x = A\dot{x} + Bu$$

To complete the state space we must define the output of interest. Suppose we want to know the voltage across the capacitor; then the output will be a state and the matrix will be:

$$C = (0\ 1) \rightarrow y = Cx = v_C$$

If we were interested to the inductor voltage, we had to define also the matrix since:

$$v_L = V_G - v_C - I_L R \rightarrow C = (-R - 1); \qquad D = 1 \rightarrow y = Cx + Du$$

If we were interested to both the states then we would have a SIMO system with the vector of the output:

$y = Cx$; $C = [0 \quad 1]$; D = [0];

Calculation: In order to calculate our system matrix, we place values of resistor, inductance and capacitor our matrix will be:

$$A=\begin{pmatrix} -\frac{50}{1*10^{-6}} & -\frac{1}{1*10^{-6}} \\ \frac{1}{5*10^{-9}} & 0 \end{pmatrix},$$

$$B=\begin{pmatrix} \frac{1}{1*10^{-6}} \\ 0 \end{pmatrix}$$

According to formulation, our output matrix will be:

$C = [0 \quad 1]$; D = [0];

Response of RLC Circuit through MATLAB:

```
clear all
close all
clc
% values of elements
R=50;
C=5e-9;
L=1e-6;
%initial conditions
iL=0;
Vc=0;
%State variables
A=[-R/L  -1/L; 1/C  0]
B=[1/L; 0]
C=[0 1]
D=[0]
G=ss(A, B, C, D)
```

```
%outputresponse of initial condition
impulse(G)
figure,
initial(G,[iL,Vc]);
[y,t,x]=initial(G,[iL,Vc]);
```

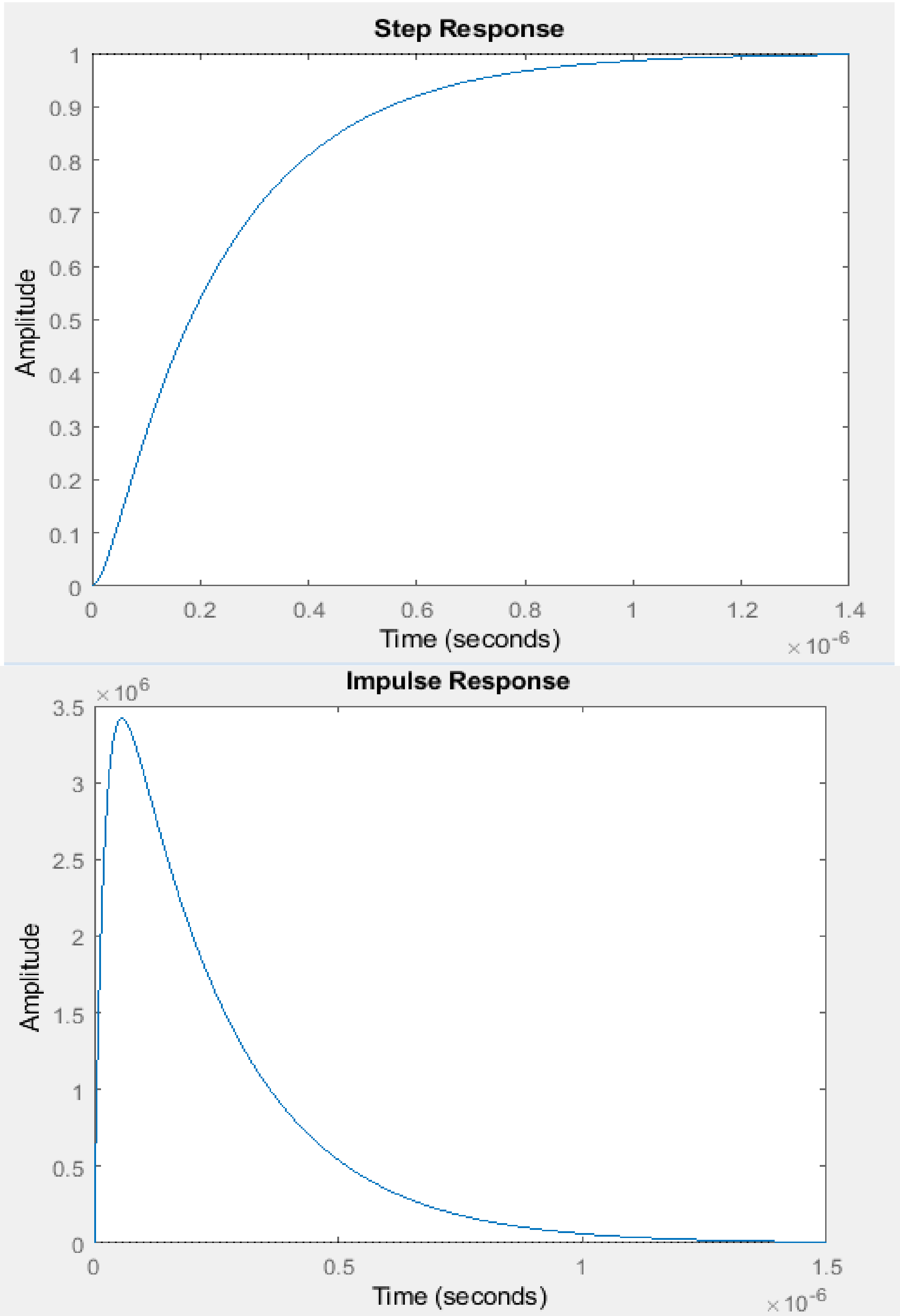

Fig.2. Output responses of the system

Conclusion: Taking into account we conclude that, by using state space method we can simply find the response and stability of the RLC circuit and also with the help of MATLAB the analysis of an RLC circuit becomes too easy. The future work is to make such algorithm which is effective and more accurate to calculate the response and analysis of an RLC circuit

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