



REGULAR ALGORITHMS FOR ESTIMATING UNCERTAIN PERTURBATIONS IN PROBLEMS OF SYNTHESIS OF INVARIANT CONTROL SYSTEMS

Buronov B. M. Senior teacher, Department of Automation and Control, Navoi State Mining Institute. Email: bunyod.buronov@inbox.ru

Abstract. Regular algorithms for estimating uncertain perturbations in the problems of synthesis of invariant control systems are given. Recurrent algorithms for estimating uncertain perturbations in dynamic control systems are proposed based on the methods of dynamic filtering and solving ill-posed problems. The regularization parameter is recommended to be determined based on the method of model examples. The above algorithms make it possible to find a regularized estimate of the input vector compatible with the observed system output, and thereby improve the accuracy of estimating the input uncertain perturbation.

Keywords: estimation of uncertain disturbances, invariant control systems, dynamic filtering, regularization, regularization parameter.

Annotatsiya. Invariant boshqaruv tizimlarini sintez qilish vazifalarida noaniq buzilishlarni baholash uchun muntazam algoritmlar berilgan. Dinamik filtrlash usullari va noto'g'ri muammolarni hal qilish asosida dinamik boshqaruv tizimlarida noaniq tartibsizliklarni baholash uchun takroriy algoritmlar taklif etildi. Tartibga solish parametr model misollar usuli asosida aniqlash uchun tavsiya etiladi. Yuqoridaqgi algoritmlar kuzatilgan tizim chiqishi bilan mos keladigan kirish vektorining tartibga solingan bahosini topishga imkon beradi va shu bilan kirish noaniqligini baholashning aniqligini oshiradi.

Kalit so'zlar: Noaniq g'alayonlarni baholash, invariant boshqarish tizimlar, dinamik filtrlash, rostlagichlar, rostlagich parametrlari.

Аннотация. Приводятся регулярные алгоритмы оценивания неопределенных возмущений в задачах синтеза инвариантных систем управления. Предложены рекуррентные алгоритмы оценивания неопределенных возмущений в динамических системах управления на основе методов динамической фильтрации и решения некорректных задач. Параметр регуляризации рекомендуется определять на основе способа модельных примеров. Приведенные алгоритмы позволяют находить регуляризованные оценку вектора входов, совместимого с наблюдаемым выходом системы, и тем самым повысить точность оценивания входного неопределенного возмущения.

Ключевые слова: оценивание неопределенных возмущений, инвариантные системы управления, динамическая фильтрация, регуляризация, параметр регуляризации.

To date, in the theory of adaptive systems, methods of parametric adaptation have received significant development [1-4]. However, the methods of signal adaptation, due to the incompleteness of the a priori and the presence of

the current uncertainty of knowledge about external influences, have not received due development, in particular, these are the issues of estimating uncertain disturbances and uncontrolled input actions in dynamic control systems. The problem of restoring the initial state and the input action of a dynamic system based on the results of measuring the output belongs to the class of inverse problems of the dynamics of controlled systems [5]. Since this problem is ill-posed, it is necessary to apply the methods developed in the corresponding theory [6,7] to solve it.

Consider a linear dynamical system with observation:

$$x_{k+1} = A_k x_k + B_k w_k, x(k_0) = x^0, \quad (1)$$

$$y_k = C_k x_k + D_k w_k, \quad (2)$$

где $x \in R^n, w \in R^p, y \in R^m$; $x = x_k$ – state of the system; x^0 – initial state of the system; $w_k \in L_2^p$ – input unmeasured perturbing effect on the system; $y_k \in L_2^m$ – system output; A_k, B_k, C_k, D_k – matrices of corresponding dimensions.

Let

$$\Theta = R^n \times L_2^p, Y = L_2^m.$$

Let us turn the space into a Hilbert space by defining the scalar product on it $\langle \theta_1, \theta_2 \rangle_\Theta = \langle x_1^0, x_2^0 \rangle_{R^n} + \langle w_1, w_2 \rangle_{L_2^p}$.

Relations (1), (2) define the linear operator $F: \Theta \rightarrow Y$, which each pair $\theta = (x_0, w) \in \Theta$, that is, the input of the system, assigns the function $y \in Y$ at the exit of the system. Let y^* – some system output (1), (2). Denote by Θ^* non-empty set of all inputs $\theta \in \Theta$ such, that

$$F\theta = y^*. \quad (3)$$

Let us consider the problem of approximate restoration of an element $\theta^* = (x_*, w^*)$ based on the results of inaccurate measurements of the output, assuming that the matrices A, B, C, D known exactly.





To solve the equation (3) we will use the concepts of dynamic filtering. To denote equation (3), we write it in the form:

$$\theta_{k+1} = \theta_k + w_k, \theta(0) = \theta_0, \quad (4)$$

$$y_{k+1}^* = F_{k+1}\theta_{k+1} + v_{k+1} (k = 0, 1, \dots), \quad (5)$$

where θ_k – system state vector, y_k^* – measurement

Vector, w_k и v_k – Gaussian white noise with zero mean and intensities Q_k , R_k ; θ_0 – Gaussian random vector with known characteristics $M(\theta_0)$ и $M(\theta_0\theta_0^T) = P_0$.

We will assume that w_k and v_k not correlated with θ_0 , but

$$M[w_k v_j^T] = S_k \delta_{kj}, S_k \neq 0,$$

where δ_{kj} - Kronecker symbol. Matrix R_k - positive definite.

Let's also assume that

$$w_k = v_k + w_k^0,$$

where

$$M[w_k^0 v_j^T] = 0,$$

$$\text{at } \forall k, j, M[w_k^0 w_j^{0T}] = Q_k^0 \delta_{kj}.$$

According to [10], we have

$$\hat{\theta}_{k+1|k} = \hat{\theta}_{k|k} + M[w_k|y_k]. \quad (6)$$

It can be shown that in the case under consideration the condition

$$M[w_k^0 y_j^{*T}] = \begin{cases} 0, & \text{если } j < k, \\ S_k, & \text{если } j = k. \end{cases} \quad (7)$$

Based on the properties of conditional mathematical expectations [8], as well as relation (7), we obtain

$$M[w_k|y_k] = W_k[y_k^* - F_k \hat{\theta}_{k|k-1}], \quad (8)$$

where

$$W_k = S_k[F_k P_{k|k-1} F_k^T + R_k]^{-1}, \quad (9)$$

$P_{k,j}$ - correlation matrix of estimation error $\varepsilon_{k|j} = \theta_k - \hat{\theta}_{k|j}$.

Substituting (8) into (6), we find

$$\hat{\theta}_{k+1|k} = \hat{\theta}_{k|k} + W_k[y_k^* - F_k \hat{\theta}_{k|k-1}]. \quad (10)$$

Based on the representations [9,10], we express $\hat{\theta}_{k+1|k+1}$ in $\hat{\theta}_{k+1|k}$:

$$\begin{aligned} \hat{\theta}_{k+1|k+1} &= \hat{\theta}_{k+1|k} + K_{k+1} \tilde{y}_{k+1|k}^*, \\ K_{k+1} &= P_{k+1|k} F_{k+1}^T G_\alpha(\mathbf{P}_{k+1}), \quad (11) \\ G_\alpha(\mathbf{P}_{k+1}) &= [\mathbf{P}_{k+1} + \alpha I]^{-1}, \\ \mathbf{P}_{k+1} &= F_{k+1} P_{k+1|k} F_{k+1}^T + R_{k+1}, \\ \tilde{y}_{k+1|k}^* &= F_{k+1} \tilde{\theta}_{k+1|k} + v_{k+1}, \\ \tilde{\theta}_{k+1|k} &= \theta_{k+1} - \hat{\theta}_{k+1|k}, \end{aligned}$$

$$y_{k+1}^* - F_{k+1} \hat{\theta}_{k+1|k} = [F_{k+1} P_{k+1|k} F_{k+1}^T R_{k+1}^{-1} + I][y_{k+1}^* - F_{k+1} \hat{\theta}_{k+1|k+1}],$$

где $G_\alpha(\mathbf{P}_{k+1})$ – generating system of functions for the regularization method, α – regularization parameter.

The regularization parameter α in (11) should be determined based on the method of model examples [11].

Then

$$\hat{\theta}_{k+1|k} = \hat{\theta}_{k|k} + D_k[y_k^* - F_k \hat{\theta}_{k|k}], \quad (12)$$

where

$$D_k = S_k R_k^{-1}.$$

By virtue of [12,13], the optimal current estimate of the state vector is determined using the relation:

$$\begin{aligned} \hat{\theta}_{k+1|k+1} &= M[\theta_{k+1}|y_1^*, \dots, y_k^*, y_{k+1}^*], \\ M[\theta_{k+1}|y_1^*, \dots, y_k^*, y_{k+1}^*] &= M[\theta_{k+1}|y_1^*, \dots, y_k^*] + \\ &\quad M[\theta_{k+1}|\tilde{y}_{k+1|k}^*], \\ \tilde{y}_{k+1|k}^* &= y_{k+1}^* - M[y_{k+1}^*|y_1^*, \dots, y_k^*], \\ \hat{\theta}_{k+1|k+1} &= \hat{\theta}_{k+1|k} + M[\theta_{k+1}|\tilde{y}_{k+1|k}^*]. \quad (13) \end{aligned}$$

Now, using equations (12) and (13), we can obtain the optimal filter equation:

$$\begin{aligned} \hat{\theta}_{k+1|k+1} &= A_k^0 \hat{\theta}_{k|k} + K_{k+1}[y_{k+1}^* - F_{k+1} A_k^0 \hat{\theta}_{k|k}] + [I \\ &\quad - K_{k+1} F_{k+1}] D_k y_k^*, \hat{\theta}_{0|0} = 0, \end{aligned}$$

where

$$A_k^0 = I - D_k F_k.$$

Based on (4) and (12), we can write:

$$\varepsilon_{k+1|k} = A_k^0 \varepsilon_{k|k} + \Gamma_k v_k,$$

where



$$\Gamma_k = (I, -D_k); v_k = (w_k^T, v_k^T)^T,$$

At that

$$L_k = M[v_k v_k^T] = \begin{pmatrix} Q_k & S_k \\ S_k^T & R_k \end{pmatrix}, \quad \Gamma_k M[v_k \varepsilon_{k|k}^T] = 0.$$

Then

$$\begin{aligned} P_{k+1|k} &= A_k^0 P_{k|k} A_k^{0T} + \Gamma_k L_k \Gamma_k^T, \\ P_{k|k} &= [I - K_k F_k] P_{k|k-1}, P_{0|0} = P_0. \end{aligned} \quad (14)$$

$$P_{k+1|k} = A_k^0 P_{k|k-1} A_k^{0T} - A_k^0 P_{k|k-1} F_k^T G_\alpha(\mathbf{P}_{k+1}) F_k P_{k|k-1} A_k^{0T} + \Gamma_k L_k \Gamma_k^T. \quad (15)$$

Equation (15) is a discrete Riccati equation, the methods of investigation of which are given, for example, in [14-20]. The matrix arises as a natural notation for the transfer matrix in equation (12). Note that the assumption that is not essential and can be taken into account by introducing the initial condition of the optimal filter equation:

$$\hat{\theta}_{0|0} = M(\theta_0).$$

The constructed algorithm generates an estimate

$$\hat{\theta}_{k|k} = M(\theta_k | y_i^*, 0 \leq i \leq k)$$

by processing current measurements y_k^* together with previous measurements y_{k-1}^* .

From equations (10) and (13) we find

$$\hat{\theta}_{k+1|k} = \hat{\theta}_{k|k-1} + K_k^0 [y_k^* - F_k \hat{\theta}_{k|k-1}],$$

where

$$K_k^0 = K_k + W_k,$$

or by virtue of (9) and (11):

$$K_k^0 = [P_{k|k-1} F_k^T + S_k] G_\alpha(\mathbf{P}_{k+1}).$$

Then we get

The above algorithms allow us to find a regularized estimate of the input vector $\theta = (x_0, w) \in \Theta$, compatible with the observed output of the system, and thereby improve the accuracy of estimating the input uncertain perturbation. Based on the solution of model examples, the consistency of the desired estimates with the properties of asymptotic convergence is shown. The practical implementation of the above algorithms under the conditions of a specific technological object, in combination with adaptive identification and control algorithms based on the theory of dynamic estimation, have shown their effectiveness [20-25].

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