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QOVUSHQOQ ELASTIK MUHITDAGI GARMONIK TO‘LQINNING PARALLEL QUVURLARDAGI KO‘P KARRALI SOCHILISHI

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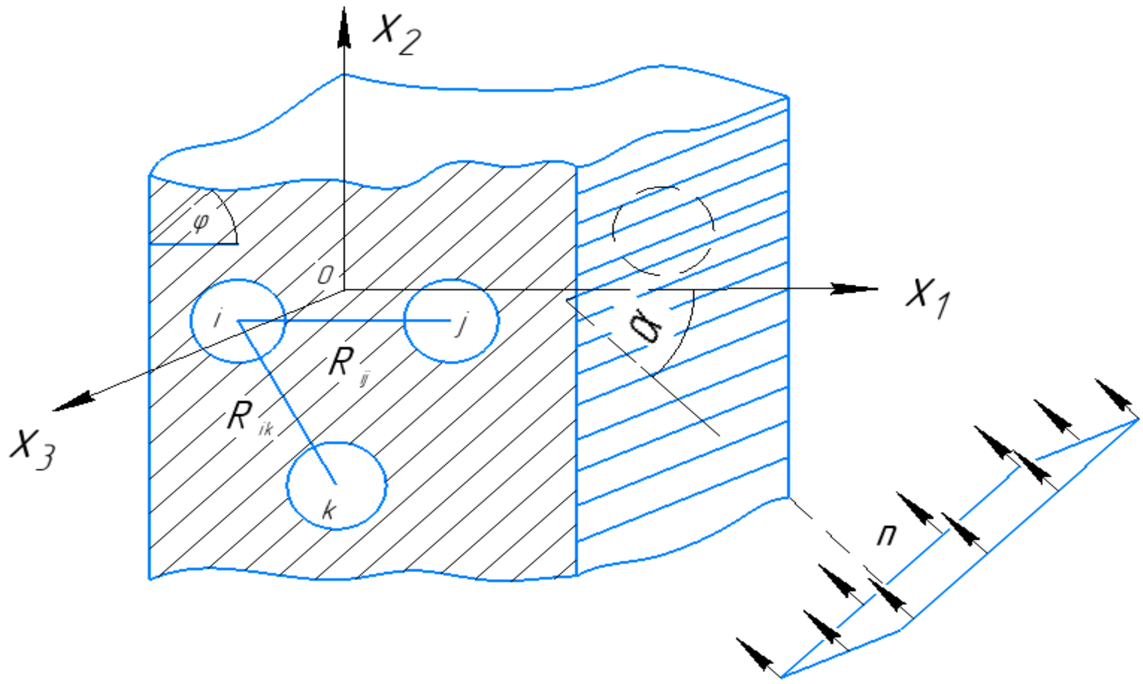
Annotatsiya. Ushbu maqolada qovushqoq-elastik muhitga bir-biriga parallel silindrik formadagi quvurlargadi garmonik to‘lqinning ko‘p karrali sochilishi aniqlanadi. Bunda to‘lqinning sochilishi silindrik funksiyalar yordamida topiladi.
Kalit so‘zlar: muhit, qovushqoq-elastiklik, garmonik to‘lqin, parallel quvurlar, kuchlanish, deformatsiya holati, silindrik bo‘shliq

Аннотация. В статье описывается многократное рассеяние гармонической волны от параллельных цилиндрических трубок в вязкоупругой среде. В этом случае рассеяние волн находится с помощью цилиндрических функций.
Ключевые слова: среда, вязко упругость, гармоническая волна, волновая нагрузка, контурные напряжения, цилиндрическая полость, деформируемая среда.

Abstract. The article describes multiple scattering of a harmonic wave from parallel cylindrical tubes in a viscoelastic medium. In this case, the scattering of waves is found using cylindrical functions.
Keywords: environment, viscous elasticity, harmonic wave, wave load, contour stresses, cylindrical cavity, deformable environment.

Kirish

Bizga qovushqoq-elastik muhitda bir-biriga parallel silindrik formadagi quvurlar berilgan bo‘lsin (1-rasm).



1-rasm. Silindrik bo‘shliqning to‘lqin bilan o‘zaro ta’siri hisob sxemasi.

Bu silindrga garmonik uzun to‘lqin tushsin. Tushadigan to‘lqin quyidagi ko‘rinishda bo‘lsin:

$$\Phi_i = \phi_0 e^{i\alpha x - i\omega t}, \tag{1}$$

Bunda, ϕ_0 – tushadigan to‘lqin amplitudasi, α – to‘lqin soni, ω – tushadigan to‘lqin chastotasi $\alpha = \omega / c_p, c_p$ – bo‘ylama to‘lqin tezligi. Silindrlar sistemasidan ajratib olingan n tartibli silindrga tushadigan to‘lqin quyidagi ko‘rinishda bo‘ladi:

$$\Phi_i = \phi_0 e^{i\alpha(\vec{r}_{0n}+\vec{r}_n) - i\omega t} = \phi_0 e^{i\alpha r_{0n}\cos(\theta_{0n}+\gamma)} \sum_{m=-\infty}^{\infty} J_m(\alpha r_n) e^{im(\theta_n+\gamma+\pi/2)-i\omega t}, \tag{2}$$

bu yerda, J_m - m-tartibli Bessel funksiyasi, ϕ_0 - amplituda, ω - aylanma chastota. To‘lqinning *k*- tartibli sochilishi silindrik funksiyalar yordamida topiladi:

$$\begin{aligned} {}^n\Phi^k &= \sum_{m=-\infty}^{\infty} {}^n A_m^k H_m(\alpha r_n) e^{im(\theta_n+\gamma+\pi/2)-i\omega t}, \\ {}^n\Psi^k &= \sum_{m=-\infty}^{\infty} {}^n B_m^k H_m(\beta r_n) e^{im(\theta_n+\gamma+\text{э})-i\omega t}, \text{э} = \pi / 2n. \end{aligned} \tag{3}$$

Bunda, H_m - Xankel funksiyasi, bir jinsli m-tartibli, $\beta = \omega / c_s$. Agar *m*-juft bo‘lib, $m > 0$ bo‘lganda, $\text{э} = 0$ bo‘ladi. Agar *m*-toq bo‘lib, $m < 0$ bo‘lsa, u holda $m = 0$ bo‘lganda, ${}^n B_m^k = 0$ bo‘ladi. Yuqorida olingan munosabatlardan foydalanib, bo‘ylama va ko‘ndalang to‘lqin potensiallarini topish mumkin

$$\begin{aligned} \Phi &= \sum_{n=0}^{N} \sum_{k=1}^{\infty} \sum_{m=-\infty}^{\infty} {}^n A_m^k H_m(\alpha r_n) e^{im(\theta_n+\gamma+\pi/2)-i\omega t}, \\ \Psi &= \sum_{n=0}^{N} \sum_{k=1}^{\infty} \sum_{m=-\infty}^{\infty} {}^n B_m^k H_m(\beta r_n) e^{im(\theta_n+\gamma+\text{э})-i\omega t}. \end{aligned} \tag{4}$$

Bunda, ${}^n A_m^k$ va ${}^n B_m^k$ - ixtiyoriy o‘zgarmaslar bo‘lib, chegaraviy shartlardan topiladi. Agar muhit n - dona silindrik bo‘shliq (yoki teshik)dan iborat bo‘lsa, u holda quyidagicha chegaraviy shart qo‘yiladi:

$$\sigma_{rr}^k \Big|_{r_n=a_n} = 0, \tau_{r\theta}^k \Big|_{r_n=a_n} = 0, k = 1,2,..; n = 0,1,2..N. \tag{5}$$

Yuqorida keltirilgan (4) ifodalardagi ${}^n A_m^k$ va ${}^n B_m^k$ ixtiyoriy o‘zgarmaslar quyidagicha topiladi:

$${}^n A_m^1 = -\phi_0 e^{i\alpha r_{0n}\cos(\theta_{0n}+\gamma)n} A_m, {}^n B_m^1 = -\phi_0 e^{i\alpha r_{0n}\cos(\theta_{0n}+\gamma)n} A_m,$$

bunda,

$${}^n A_m^k = [D_{ma}K_{ma} - E_{ma}H_{ma}] / [D_{ma}K_{ma} - EE_{ma}H_{ma}] \text{ va}$$

$${}^n B_m^k = [D_{ma}EE_{ma} - E_{ma}DD_{ma}] / [DD_{ma}K_{ma} - EE_{ma}H_{ma}],$$

$$D_{ma}=(m^2+m-\frac{1}{2}\beta^2a^2)J_m(\alpha a)-\alpha aJ_{m-1}(\alpha a);$$
$$E_{ma}=m(m+1)J_m(\alpha a)-m\alpha aJ_{m-1}(\alpha a);$$
$$H_{ma}=-m(m+1)J_m(\beta a)-m\beta aJ_{m-1}(\beta a);$$
$$K_{ma}=-(m^2+m-\frac{1}{2}\beta^2a^2)J_m(\beta a)+\beta aJ_{m-1}(\beta a);$$
$$DD_{ma}=(m^2+m-\frac{1}{2}\beta^2a^2)H_m(\alpha a)-\alpha aH_{m-1}(\beta a);$$
$$EE_{ma}=m(m+1)H_m(\alpha a)-m\alpha aH_{m-1}(\alpha a);$$
$$HH_{ma}=-m(m+1)H_m(\beta a)-m\beta aH_{m-1}(\beta a);$$
$$K_{ma}=-(m^2+m-\frac{1}{2}\beta^2a^2)H_m(\beta a)+\beta aJ_{m-1}(\beta a).$$

Ixtiyoriy oʻzgarmaslar ${}^{n}A_m^k$ va ${}^{n}B_m^k$ uchun quyidagi rekurent munosabatni olamiz [1]. Agar bitta silindrik formadagi boʻshliq berilsa (r = *a*), unga tekis (1) koʻrinishdagi toʻlqin tushsa, u holda uning konturida hosil boʻladigan eng katta kuchlanish kontur kuchlanishi boʻladi. Agar biror silindrik formadagi Qattiq jism qoʻyilsa, u holda konturdagi maksimal kuchlanishlar $\sigma_{\theta\theta}^*,\sigma_{rr}^*$ boʻladi. Bu kuchlanishlarning silindrik formadagi qattiq jism chegarasidagi ifodasi quyidagicha boʻladi:

$$\sigma^*_{rr}=-\frac{\eta}{\pi}\sum_{m=0}^{\infty}\frac{\in_n i^{n+1}I_n(\bar{\alpha}_2\ a)\bar{\beta}_1^{\,2}a^2}{\Delta_n}\times\left\{\beta_1 aH_{n-1}^{(1)}(\bar{\beta}_1 a)-(n^2+n-\frac{\bar{\beta}_1^{\,2}\ a^2}{2}\ H_n^{(1)}\ (\bar{\beta}_1\ a)\ \right\}\cos\quad ne^{-i\omega t}$$

$$\sigma^*_{r\theta}=-\frac{\eta}{\pi}\sum_{n=0}^{\infty}\frac{\in_n i^{n+1}}{\Delta n}\left\{\left[(\frac{1}{\bar{\chi}^2}-1)(\frac{n\bar{\beta}_1^2a^2}{2}(n-1)-n^2(n^2-1)-\eta\frac{\bar{\beta}_1^4a^4}{4})+\frac{\bar{\beta}_1^2a^2}{4}\eta(n(n+1)-\frac{1`}{2}\bar{\beta}_1^2a^2)\right]\cdot\right.$$
$$\left.\cdot I_n(\alpha_2 a)+\left(\frac{1}{\chi^2}-1\right)\left(n^3-n+\frac{1}{2}\bar{\beta}_1^2a^2\right)(\alpha_2 a)I_{n-1}(\alpha_2 a)H_n^{(1)}(\bar{\beta}_1\ a)+\left(\frac{1}{\chi^2}-1\right)(1-n^2)(\alpha_2\bar{\beta}a^2)I_{n-1}(\alpha_2 a)H_{n-1}(\bar{\beta}_1\ a)\ \right\}\cos n\theta e^{-i\omega t},$$

$$\eta=\rho_2/\rho_1\quad\bar{\chi}^2=\frac{\bar{\beta}_1^2}{\bar{\alpha}_1^2}.$$

Haqiqiy argumentning Bessel va Xankel funksiyalari xuddi logarifmning trigonometrik funksiyalari kabi jadvalga kiritilgan va ular argumentining ma’lum kattaliklarida hisoblanadi. Bu haqiqiy funksiyalar boʻlgani uchun ular statsionar toʻlqinlarni tavsiflaydi. Aslida statsionar silindrsimon harakatlanuvchi toʻlqinning superpozitsiyasidir: biri silindr oʻqidan ajralib chiqadi, ikkinchisi esa cheksizlikdan silindr tomon yaqinlashadi. r=0 da bu ikki toʻlqinning fazalari qarama-qarshi boʻlganligi uchun ular bir-birini bekor qiladi va shuning uchun statsionar toʻlqinning amplitudasi r=0 da chekli boʻlib qoladi. Bu gap qovushqoq-elastik toʻlqinning tarqalishi masalasini hal qilishda qoʻllaniladi. Silindrsimon toʻlqin I_0 (kr) tekislik statsionar toʻlqinga mos keladi cos(kx-π /4). Energiyaning tobora kattaroq silindrsimon sirtlarga taqsimlanishi tufayli, masofaning ortishi bilan kamayib boradi. Silindrsimon toʻlqin N_0(kr) asimptotik ravishda tekis toʻlqin sin (kx-π/4)ga mos keladi. Silindr oʻqida (r=0) tushayotgan va qaytadigan toʻlqinlarning grad u bir xil boʻlib, toʻlqinning amplitudasi tekis toʻlqin holatidan farqli ravishda cheksiz katta boʻladi; shuning uchun silindr oʻqidagi N_0 funksiya (r=0) uzilishga ega.

Agar tushadigan bo‘ylama (yoki ko‘ndalang) to‘lqin (1) uzoq vaqtda chesizlikdan kelsa, u holda bitta mustahkamlanmagan (silindrik formadagi bo‘shliq uchun) cheksizlikda quyidagi statik yechim olinadi va quyidagicha bo‘ladi (siljish deformatsiyasi uchun):

$$\sigma^*_{\theta\theta} = -4\sin 2\theta \, .$$

Silindrik formadagi mustahkamlanmagan bo‘shliq uchun statik yechim quyidagicha bo‘ladi (α 0):

$$\sigma^*_{\theta\theta} = \frac{2}{\chi^2}\left[\left(\overline{\chi}^2 - 1\right) - 2\cos 2\theta\right].$$

Agar muhitda silindrik formadagi absolyut qattiq jism berilgan bo‘lsa, u holda statik yechim quyidagicha bo‘ladi:

$$\sigma^*_{rr} = -\frac{2\overline{\chi}^2}{\chi^2 + 1}\sin 2\theta;$$

$$\sigma^*_{r\theta} = -\frac{2\overline{\chi}^2}{\chi^2 + 1}\cos 2\theta;$$

$$\sigma^*_{\theta\theta} = (1 - \frac{2}{\chi^2})\sigma^*_{rr}.$$

Bessel funksiyasi argumentining kichik qiymatlarida qatorning birinchi hadini olish bilan cheklanish mumkin. Agar garmonik to‘lqin silindrik formadagi jismga tushsa, u holda $\sigma^*_{\theta\theta}$ kontur kuchlanishi β 0 va α 0 hamda n = 1 nolga intilsa, n ≥2 uchun quyidagi munosabat o‘rinli bo‘ladi:

$$\frac{n\,(n^2 - 1 - \dfrac{\overline{\beta}^2 a^2}{2} H_n(\alpha a))}{\Delta_n} \to \begin{cases} \dfrac{i\pi}{2\left(1 - \dfrac{1}{\chi^2}\right)} & n = 2, \\ 0 & n > 2. \end{cases}$$

Qachonki, η = 0 va n=1 bo‘lganda, β0 intilsa, u holda Xankel funksiyasini quyidagicha tasvirlash mumkin:

$$\frac{H_1(\beta a)}{\Delta_1} \approx \frac{\overline{\chi}^2}{(1+\overline{\chi}^2)\overline{\beta} a} = \frac{\overline{\chi}^2}{(1+\overline{\chi}^2)\beta(1-\Gamma_R)}.$$

Xankel funksiyasining αa>> 1 asimptotik ifodasi quyidagicha bo‘ladi:

$$H_n(\alpha a) \sim \left(\frac{2}{\pi\alpha a}\right)^{1/2} e^{i(\alpha a - n\pi/2 - \pi/4)}.$$

Qattiq jismning ko‘chishi (bo‘ylama to‘lqin ta’sirida) quyidagi ko‘rinishni egallaydi:

$u = \eta a^{-1}\left[2i\varphi_0 I_1(\alpha a) + A_1 H_1(\alpha a) + B_1 A_1(\beta a)\right]$, bunda, $\eta = \rho_1 / \rho_2$.

Yuqoridagi ifodalar va chegaraviy shartlardan foydalansak, kompleks koeffitsientli algebraik tenglamalar sistemasini olamiz, uni Gauss usuli yordamida yechamiz:

$$A_n = -\frac{\varphi_0 \in_n i^n}{\Delta n}\left[\bar{\alpha}\bar{\beta}a^2 I_n'(\bar{\alpha}a)H_n'(\bar{\beta}a) - n^2 I_n(\bar{\alpha}a)H_n(\bar{\beta}a)\right];$$

$$B_n = -\frac{\varphi_0 \in_n i^{n+1}}{\Delta n}\left(\frac{2\xi n}{\pi}\right)$$

$$\Delta n = \left[\bar{\alpha}\bar{\beta}a^2 H_n'(\bar{\alpha}a)H_n'(\bar{\beta}a) - n^2 H_n(\bar{\alpha}a)H_n(\bar{\beta}a)\right], \text{для} \quad n=1$$

$$A_1 = \frac{2i\varphi_0}{\Delta}[-4\eta I_1(\bar{\alpha}a)H'(\bar{\beta}a) + (1+\eta)I_1(\bar{\alpha}a) * \bar{\beta}aH_0(\bar{\beta}a) +$$

$$+(1+\eta)\bar{\alpha}aI_0(\bar{\alpha}a)H_1(\bar{\beta}a) - \bar{\alpha}\bar{\beta}a^2 I_0(\bar{\alpha}a) * H_0(\bar{\beta}a)];$$

$$B = -\frac{2i\varphi_0}{\Delta_1}\cdot\frac{2i(1-\eta)}{\pi};$$

$$\Delta_1 = 4nH_1(\bar{\alpha}a)H_1(\bar{\beta}a) - (1+\eta)\bar{\beta}aH_0(\bar{\beta}a)H_1(\bar{\alpha}a) -$$

$$-(1+\eta)\bar{\alpha}aH_0(\bar{\alpha}a)H_1(\bar{\beta}a) + \bar{\alpha}\bar{\beta}a^2 H_0(\bar{\alpha}a) * H_0(\bar{\beta}a).$$

Uzun to‘lqinlarda qattiq jism ko‘chishining asimptotik ifodasi quyidagicha bo‘ladi:

$$u*(\bar{\alpha}a) \approx \frac{\bar{\alpha}^2 a^2}{4\eta}[\chi^2(\eta-1)\ln(\chi\bar{\alpha}a) + \ln\bar{\alpha}a] + \frac{i\pi\ \alpha^2 a^2}{8\eta}[(1-\eta)\chi^2 + (1+\eta)];$$

$$\lim_{\alpha a \to 0} u^*(\alpha a) = 1.$$

Agar deformatsiyalanuvchi muhit silindrik bo‘shliq yupqa qobiq bilan mustahkamlangan bo‘lsa, u holda COS (nθ) va SIN (nθ) burchak bo‘yicha trigonometrik formada yozib olinadi. Bu yerda, n - butun son bo‘lib, 0 dan ∞ gacha o‘zgaradi. Agar n=0 bo‘lsa, mustahkamlangan silindrik bo‘shliq sirtida grunt bosimi teng taqsimlanadi. Xuddi shunday n=1 bo‘lganda, mustahkamlangan silindrik bo‘shliq, qattiq jism kabi harakatda bo‘ladi. Tebranishlar formasi n>2 bo‘lganda, mustahkamlangan silindrik bo‘shliq va uni o‘rab turuvchi muhitning deformatsiyasi hisobga olinadi. Shu holatlarni qisqacha ko‘rib chiqamiz.

Xususiy holda n=0 bo‘lgan holni ko‘ramiz. Og‘ma toyilishlar va qo‘shimcha potensiallar formulalar bilan ifodalangan:

$$\varphi_{над}^{(p)} = P_0 J_0(k_1 r); \quad (P_0 = 1) \quad , \phi_{отр}^{(s)} = A_0 H_0(k_1 r), \ \psi_{отр} = 0.$$

Elastik muhit ko‘chishlari tekis deformatsiya holati uchun quyidagi ko‘rinishni egallaydi:

$$U_r = \frac{\partial\varphi}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}; U_\theta = \frac{1}{r}\frac{\partial\varphi}{\partial\theta} + \frac{\partial\psi}{\partial r}; \tag{6}$$

bunda, n=o; U_r va U_θ quyidagi ko‘rinishni egallaydi:

$$U_r = \frac{\partial\varphi}{\partial r} = -k\left[J_1(k_1 r) + L_0 H_1^{(2)}(k_1 r)\right]_{r=r_0}; \quad U_\theta = 0;$$

Radial kuchlanish quyidagi formula bilan aniqlanadi:

$$\frac{\sigma_{rr}}{2\mu} = 0.5\beta^2\varphi + \frac{1}{r}\left[\frac{\partial\varphi}{\partial r} + \frac{1}{r}\frac{\partial^2\varphi}{\partial\theta^2} + \frac{1}{r}\frac{\partial\varphi}{\partial\theta} - \frac{\partial^2\varphi}{\partial r\partial\theta}\right]$$

$$\frac{\sigma_{zz}}{2\mu} = \left(\alpha^2 - 0.5\beta^2\right)\phi. \tag{7}$$

bunda, $\beta = \omega / c_s; \alpha = \omega / c_p$.

Atrof-muhitning radial kuchlanishi to‘lqinlar o‘tishi bilan quyidagi ko‘rinishni oladi:

$$\sigma_r = -\rho a^2 K_1^2 \left[J_0(k_1 r) + A_0 H_0^{(2)}(k_1 r) \right] + \frac{2\rho b^2}{r} k_1 \left[J_1(k_1 r) + L_0 H_0^{(2)}(k_1 r) \right]_{r=r_0}; \qquad (8)$$

$$\sigma_{r\theta} = 0.$$

Mustahkamlangan bo‘shliq radial ko‘chish tekis taqsimlangan yuklanish ostida quyidagicha bo‘ladi:

$$U_R = \left. \frac{r^2 q}{hE_K} \right|_{r=r_0}; \qquad (9)$$

bu yerda, q - quvurga tushadigan tashqi bosim; h - quvur qalinligi, E_K - materialning elastiklik moduli. Xususiy holda n=1 bo‘lgan holni ko‘ramiz. Quvurga tushadigan to‘lqin n=1 bo‘lganda, quyidagi ko‘rinishni ifodalaydi:

$$\phi_{na\partial}^{(p)} = 2ij_1(k_1 r)\cos\theta; \qquad (10)$$

Agar bo‘shliq Qattiq jism bilan mustahkamlangan bo‘lsa, u holda muhitga qaytgan to‘lqin potensiallari (n=1 uchun) quyidagi ko‘rinishni egallaydi:

$$\phi^{(s)} = A_1 H_1^{(1)}(k_1 r)\cos\theta;\ \psi^{(s)} = B_1 H_1^{(1)}(k_1 r)\sin\theta.$$

Ko‘chish potensiallari yordamida muhit va quvurning ko‘chishi hamda kuchlanishlarini aniqlash mumkin:

$$U_{r1} = \left[k_1 H_1^{(2)}(k_1 r_0) A_1 + \frac{1}{r_0} H_1^{(2)}(k_1 r_0) B_1 + \phi_0 i k_1 J_1(k_1 r_0) \right] \cos\theta \qquad (11)$$

$$U_{\theta 1} = -\left[\frac{1}{r_0} H_1^{(2)}(k_1 r_0) A_1 + k_2 H_1^{(2)}(k_1 r_0)' B_1 + \frac{\phi_0 i}{G_0} J_1(k_1 r_0) \right] \sin\theta$$

$$\sigma_{r1} = A \left[-\rho a^2 k^2 H_1^{(1)}(k_1 r_0) + \frac{2\rho b^2 k_1}{r_0} H_2^{(1)}(k_1 r_0) \right] \cos\theta -$$

$$-B_1 \frac{2\rho b^2 k_2}{r_0} H_2^{(2)}(k_2 r_0)\cos\theta + \left[-\rho a^2 k_1^2 J_1(k_1 r_0) + \frac{\phi \rho b^2 k_1}{r_0} J_2(k_1 r_0) \right] \cos\theta;$$

$$\sigma_{r\theta 1} = \frac{2\rho b^2 k_1}{r_0} H_2^{(2)}(k_1 r_0) A_1 \sin\theta + B_1 \left[\rho b^2 k_2^2 H_1^{(2)}(k_1 r_0) \frac{2\rho b^2 k_2}{r_0} H_2^{(2)}(k_1 r_0) \right] \sin\theta + \frac{2\rho b^2 k_1}{r_0} J_2(k_1 r_0) \sin\theta.$$

Ixtiyoriy integral doimiylari A_n va B_n - ni topish uchun chegaraviy shartlardan foydalanamiz:

$$\sigma_{rr}\Big|_{r=r_0} = 0; \quad \sigma_{r\theta}\Big|_{r=r_0} = 0, \qquad (12)$$

A_1 va B_1 koeffitsientlar quyidagicha topiladi:

$$A_1 = \frac{\Delta A}{\Delta}; \qquad B_1 = \frac{\Delta B}{\Delta}, \qquad (13)$$

bunda, $\Delta A = -\frac{2i}{r_0}\left[\alpha J_1(\alpha)H_1^{(2)}(m\alpha) + m\alpha J_1(\alpha)H_2^{(2)}(m\alpha)\right];$

$$\Delta B = -\frac{2i}{r_0}\alpha\left[J_1(\alpha)H_2^{(2)}(\alpha) - J_2(\alpha)H_1^{(2)}(\alpha)\right]; \qquad (14)$$

Bu, Bessel funksiyalarini hisoblashda quyidagi munosabatlardan foydalanilgan:

$$J_\nu(\alpha)H_{\nu-1}^{(2)}(\alpha) - J_{\nu-1}(\alpha)H_\nu^{(2)}(\alpha) = \frac{2}{\pi i\alpha};$$

$$J_\nu(\alpha)H_\nu^{(2)}(\alpha) - J_\nu'(\alpha)H_\nu^{(2)}(\alpha) = \frac{2i}{\pi\alpha}$$

$$\alpha J_\nu'(\alpha) = \nu J_\nu(\alpha) - \alpha J_{\nu+1}(\alpha); \qquad (15)$$

$$\alpha H_\nu^{(2)'}(\alpha) = \nu H_\nu^{(2)}(\alpha) - \alpha H_{\nu+1}^{(2)}(\alpha);$$

Yuqorida keltirilgan munosabatlardan foydalanib, A_1 va B_1 koeffisientlarni topish mumkin. Bu qattiq material bilan mustahkamlangan silindrik bo‘shliq uchun olindi:

$$U_1 = \frac{\Delta A}{\Delta}k_1 H_1^{(2)'}(\alpha) + \frac{\Delta B}{r_0\Delta}H_1^{(2)}(m\alpha) + 2ik_1 J_1'(\alpha) = \frac{4}{\pi r_0\alpha} * \frac{H_2^{(2)}(m\alpha)}{H_2^{(2)}(\alpha)H_1^{(2)}(m\alpha) + mH_1^{(2)}(\alpha)H_2^{(2)}(m\alpha)} \qquad (16)$$

Ko‘chish amplitudasi quyidagi formula bilan topiladi:

$$U = \sqrt{(\mathrm{Re}U_1)^2 + (\mathrm{Im}U_1)^2}. \qquad (17)$$

Umumiy holda $n \geq 2$ bo‘lgan xolatni ko‘ramiz. Grunt muhitida joylashgan silindrik formadagi inshootni hisoblash uchun, ta’sir etuvchi tashqi kuchlar quyidagi ko‘rinishda olinadi:

$$\sigma_{rr} = \sum_{n=2}^{\infty} A_n' \cos n\theta; \quad \sigma_{r\theta} = \sum_{n=2}^{\infty} B_n' \sin n\theta. \qquad (18)$$

Yuqorida keltirilgan (18) ifodadagi A_n' va B_n' koeffisientlarga tushuvchi va qaytgan $\varphi^{(s)}, \psi^{(s)}$ to‘lqin potensiallari kiradi. Bu koeffisientlarni topishda to‘lqin tenglamasi (gruntli muhit) va quvur uchun Lame tenglamasidan foydalaniladi. Quvurga tushadigan radial bosim quyidagi ko‘rinishda olindi: $g_{n.} = A_n' d_n COSn\theta$. Bu formuladan foydalanib, quvur radial ko‘chishini topish mumkin:

$$U_{rG} = A_n' d_n COSn\theta \qquad (19)$$

bunda $d_n = \frac{r_0^n}{(n^2-1)^2 E_k J_k}\left[1 + \frac{\ell^2(acn^2-1)}{r_0^2}\right];$

r_0 - quvur radiusi; E_k, J_k - quvurning egilishdagi bikirligi; ℓ - quvurning inersiya radiusi; $a = E_k / G;$ G - ko‘ndalang kesimga bogliq bo‘lgan koeffitsient to‘g‘ri burchakli ko‘ndalang kesimli quvur uchun G =1.5. Tangensial ko‘chish quyidagi ifoda bilan aniqlanadi:

$$U_{\theta G} = -A_n' \frac{d_n}{n}\sin n\theta. \qquad (20)$$

Tangensial kuch $g_{I.} = B_n SIN\theta$ ta’sirida quvurning radial ko‘chishi quyidagicha aniqlanadi:

$$U_{rI} = -B_n{}' \frac{d_n}{n} \cos n\theta. \tag{21}$$

Quvurning normal va tangensial yuklanish ta’siridagi radial ko‘chishi quyidagi formula bilan topiladi:

$$U_{rn} = \left[A_n{}' - \frac{B_n{}'}{n} \right] d_n \cos n\theta \tag{22}$$

Xuddi shunday urinma ko‘chish quyidagicha topiladi:

$$U_{\theta n} = -\left[\frac{A_n{}' d_n}{n} - B_n{}' t_n \right] \sin n\theta \quad . \tag{23}$$

Agar quvurda hosil bo‘ladigan ko‘ndalang va o‘q bo‘yicha bo‘ladigan deformatsiyani hisobga olmasak, u holda radial va urinma ko‘chishlar quyidagicha bo‘ladi:

$$U_{rn} = \frac{r_0^4}{n\left(n^2-1\right) E_k J_k} \left[A_n - \frac{B_n}{n} \right] \cos n\theta; \tag{24}$$

$$U_{\theta n} = \frac{r_0^4}{n\left(n^2-1\right) E_k J_k} \left[A_n - \frac{B_n}{n} \right] \sin n\theta;$$

bu yerda, $A_n{}' - \frac{B_n}{n}$ kattalik (24) ko‘chish munosabatlaridan topiladi. Yuqoridagi (24) va chegaradagi kuchlanishlarning tenglik shartidan foydalansak, quyidagi munosabatni olamiz [2-4]:

$$A_n{}' - \frac{B_n}{n} = \frac{\rho b^2}{r_0^2} \left\{ A_n \left[a^2 m^2 - 2\left(n^2-1\right) \right] H_n^2(a) + B_n \frac{am}{n} * \left[amH_n^2(am) - 2\left(n^2-1\right) H_n^{2\prime}(am) \right] + 2i^n \left[a^2 m^2 - 2\left(n^2-1\right) \right] J_n(a) \right\} \tag{25}$$

Agar chegaradagi qattiq mahkamlanganlik shartidan foydalansak, u holda ixtiyoriy o‘zgarmaslar A_n, B_n ni topish uchun algebraik tenglamalar sistemasini olamiz:

$$A_n \left\{ a\mathrm{H}_n^{2\prime}(a) + \frac{\gamma}{n^2-1} \left[\frac{a^2 m^2}{n^2-1} - 2 \right] \mathrm{H}_n^2(a) \right\} + B_n *$$

$$* \left\{ nH_n^2(am) + \frac{a\gamma m}{\left(n^2-1\right)n} \left[\frac{am}{n^2-1} H_n^2(am) - 2H_n^{2\prime}(am) \right] - \right\} = \tag{27}$$

$$= -2i^n \left\{ aJ_n{}'(a) + \frac{\gamma}{n^2-1} \left[\frac{a^2 m^2}{n^2-1} - 2 \right] \mathrm{J}_n(a) \right\};$$

$$A_n \left\{ a\mathrm{H}_n^{2\prime}(a) + \frac{\gamma}{n^2-1} \left[\frac{a^2 m^2}{n^2-1} - 2 \right] \mathrm{H}_n^2(a) \right\} + B_n *$$

$$* \left\{ nH_n^2(am) + \frac{a\gamma m}{\left(n^2-1\right)n} \left[\frac{am}{n^2-1} H_n^2(ma) - 2H_n^{2\prime}(ma) \right] - \right\} =$$

$$=-2i^n\left\{aJ_n{}'(a)+\frac{\gamma}{n^2-1}\left[\frac{a^2m^2}{n^2-1}-2\right]\mathrm{J}_n(a)\right\};$$

$$A_n\left\{a\mathrm{H}_n^2(a)+\frac{\gamma}{n^2-1}\left[\frac{a^2m^2}{n^2-1}-2\right]\mathrm{H}_n^2(a)\right\}+B_n*$$

$$*\left\{naH_n^{2\,\prime}(am)+\frac{a\gamma m}{(n^2-1)n^2}\left[\frac{am}{n^2-1}H_n^2(ma)-2H_n^{2\,\prime}(ma)\right]-\right\}=$$

$$=-2i^n\left\{aJ_n(a)+\frac{\gamma}{n(n^2-1)}\left[\frac{a^2m^2}{n^2-1}-2\right]\mathrm{J}_n(a)\right\};$$

bunda,

$$\gamma=\frac{E_{cp}}{E_k2(1+\nu)}\cdot 12\left[\frac{r_0}{h}\right]^3;$$

E_{cp} - muhitning elastiklik moduli; ν - muhitning Puasson koeffisienti; h – quvur qalinligi. Noma'lum koeffitsientlar A_n va B_n quyidagi formula bilan aniqlanadi:

$$A_n=\frac{\Delta A}{\Delta};\qquad B_n=\frac{\Delta B}{\Delta};\tag{28}$$

bunda,

$$\Delta=a_nH_n^{(1)}(\alpha)H_n^{(1)}(m\alpha)+b_nH_n^{(1)\prime}(m\alpha)+c_nH_n^{(1)}(\alpha)H_n^{(1)\prime}(m\alpha)++d_nH_n^{(1)\prime}(\alpha)H_n^{(1)\prime}(m\alpha);$$

$$\Delta A=2i^na_nJ_n(\alpha)H_n^{(1)}(m\alpha)+b_nJ_n{}'H_n^{(1)\prime}(m\alpha)+c_nJ_n(\alpha)H_n^{(1)\prime}(m\alpha)+$$
$$+d_nJ_n{}'(\alpha)H_n^{(1)\prime}(m\alpha);$$

$$\Delta B=\frac{4i^{n+1}}{\pi}\left[n+\frac{\gamma}{n(n^2+1)}\left[\frac{\alpha^2m^2}{n^2-1}-2\right]\right];\quad a_n=-\frac{2\gamma m\alpha^2}{(n-1)^2}+\frac{2\gamma}{n^2-1}-n^2$$

$$b_n=\frac{\gamma m^2\alpha^2}{n^2(n^2-1)};\quad \mathrm{c}_n=\frac{\gamma m^3\alpha^3}{(n^2-1)^2};\quad d_n=-m\alpha^2\left[\frac{2\gamma}{n^2(n^2-1)}-1\right];$$

Oxirgi olingan ifodalardan kompyuterda natijalar olish uchun kompleks analiz usullari va munosabatlaridan foydalanib, qo'yilgan masala yechiladi.

Shu yuqorida keltirilgan usul va kiritilgan potensial funksiyalar yordamida bir-biriga parallel quvurlarda hosil bo'ladigan kuchlanish va deformatsiya holatini aniqlash mumkin.

Deformatsiyalanuvchi muhitda joylashgan (tekis deformatsiya holati), o'qlari bir-biriga parallel bo'lgan mustahamlangan va mustahkamlanmagan silindrik bo'shliqlarning xos tebranishlar chastotasini topish, mexanik sistemada bo'ladigan rezonans hodisalarini o'rganishga imkon yaratadi.

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