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VIBRATIONS OF A CURVED VISCOELASTIC PIPE WITH INTERNAL PRESSURE

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Abstract. The article analyzes a resolving system of equations of free oscillations of thin-walled multilayer curved composite viscoelastic pipes with internal pressure. For two types of boundary conditions, depending on internal pressure, geometric and structural factors, their proper oscillation is investigated. Taking into account the initial irregularities of the cross section and the rheological properties of the material, the dynamic state of a composite viscoelastic pipe under pressure is considered. The results of numerical analysis of the spectra of lower complex natural frequencies depending on internal pressure, geometric, structural factors, rheological properties and boundary conditions are presented. Ten primary private forms are constructed. The results of the solution are compared with known solutions and experimental results.

Keywords: fluctuations, initial irregularities of the section, boundary conditions, flexibility, viscoelastic pipe, internal pressure.

КОЛЕБАНИЯ КРИВОЛИНЕЙНОЙ ВЯЗКОУПРУГОЙ ТРУБЫ С ВНУТРЕННИМ ДАВЛЕНИЕМ

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Аннотация. В статье представлена разрешающая система уравнений свободных колебаний тонкостенных многослойных криволинейных композитных вязкоупругих труб с внутренним давлением. Для двух типов граничных условий в зависимости от внутреннего давления, геометрических и структурных факторов, исследована их собственное колебание. С учетом начальных неправильностей сечения и реологического свойства материала рассмотрено динамическое состояние композитной вязкоупругой трубы под действием давления. Приведены результаты численного анализа спектров низших комплексных собственных частот в зависимости от внутреннего давления, геометрических, структурных факторов, реологических свойств и граничных условий. Построены десять низких собственных форм. Результаты решения сопоставлены с известными решениями и результатами экспериментов.

Ключевые слова: колебания**,** начальные неправильности сечения, граничные условия, гибкость, вязкоупругая труба, внутреннее давление.

ICHKI BOSIM BILAN EGRI CHIZIQLI QAYISHQOQELASTIK QUVURLARNING TEBRANISHLARI

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Annotatsiya. Maqolada ichki bosimli yupqa devorli ko’p qatlamli kompozit qayishqoqelastik quvurlarning erkin tebranishlari tenglamalarining hal qilish tizimi tahlil qilinadi. Ikki turdagi chegara shartlari uchun, ichki bosimga, geometrik va strukturaviy omillarga qarab, ularning to’g’ri tebranishi o’rganiladi. Kesimning dastlabki nosimmetrikliklari va materialning reologik xususiyatlarini hisobga olgan holda, bosim ostida kompozit qayishqoqelastik quvurning dinamik holati ko’rib chiqiladi. Ichki bosim, geometrik, strukturaviy omillar, reologik xususiyatlar va chegara sharoitlariga qarab quyi murakkab tabiiy chastotalar spektrlarini raqamli tahlil qilish natijalari keltirilgan. O’nta quyi xususiy shakllar qurilgan. Olingan natijalar ma'lum bo'lgan yechimlar va eksperimental natijalar bilan taqqoslangan.

Kalit so'zlar: tebranishlar, kesimning dastlabki tartibsizliklari, chegara shartlari, egiluvchanlik, qayishqoqelastik quvur, ichki bosim.

Introduction.

The problem of vibrations is relevant not only for oil and gas pipelines, but also for aircraft. Pulsating flows and intense vibrations associated with them also occur in power plants [1]. Analysis of the behavior of pipelines in nuclear power plants showed [2,3] that vibrations are observed at frequencies from 0.5 to 1000 Hz, vibration amplitudes - 3-5 mm. The phenomena of pulsations in the coolant circulation circuit [4] and vibration of equipment acquire the greatest acuteness. There are cases of violation of the integrity of pipes. The nature of the fractures indicates the fatigue nature of the destruction. The origin and development of cracks is associated with elastic vibrations and hydraulic shocks. When the frequencies of the excitation spectrum coincide with the natural frequencies of the pipeline, resonance develops in the system. Obviously, this approach is justified only for thick-walled structures. The development and creation of composite structures is a more complex problem that can be effectively solved only in an inseparable unity: material -construction - technology. The subject of design becomes the material itself, or rather its structure. The new material is designed taking into account technological capabilities for a given design and a given load. Only in such unity is it possible to realize the potential inherent in the composite. However, the method of calculating the dynamic parameters of multilayer pipes, taking into account the layered-fibrous structure and anisotropy of the material, is practically absent to date. The influence of inhomogeneities of the material structure, as well as initial ovalities and different cross-sectional thicknesses on the stress state and dynamic properties of composite pipes has not been sufficiently studied.

Problem statement and solution methods

Consider a composite viscoelastic tube whose centerline represents an arc of a circle of radius R with length L. The pipe has a cross-section with a nominal average radius r and a wall thickness h. We will limit ourselves to thin-walled sufficiently long pipes of small curvature: $h/r \le 1/20, L/r \ge 4, r/R \le 1/5$. The pipe is considered as an element of the pipeline conducting the liquid. The internal flow is considered homogeneous, the liquid is single–phase, ideal and incompressible. The end sections of the pipe are closed with absolutely rigid weightless flanges, which are supported by fixed hinge supports. The boundary conditions have the form: when $s=0$ and $s=L$ $(0 \le s \le L)$ at points $\varphi_0 = 90^0$ with $\varphi_0 = 270^0$ coordinates and movements $u = \vartheta = w = 0$. To describe the viscoelastic properties of the body material, we adopt the linear hereditary Boltzmann-Voltaire theory, while we define the physical relations for the m-th viscoelastic element of the system in the form [5]

$$\sigma_{mk}^n(t) = \tilde{\lambda}_n \Theta^n(t)\delta_{mk} + 2\tilde{\mu}_n \varepsilon_{mk}^n(t) \quad (1)$$

where are $\tilde{\lambda}_n, \tilde{\mu}_n$ - Voltaire integral operators, which are replaced by one operator below.

The Poisson's ν_n ratio in the proposed formulation of the problem is assumed to be constant. This means that for a structurally inhomogeneous viscoelastic system, the forms of natural oscillations will be equal to the eigenvectors of the corresponding elastic problem [6,7].

Expressing by known formulas through $\tilde{E}_n, \tilde{\nu}_n$, and considering that $\tilde{\nu}_n = \nu_n = const$, instead of (1) we get

$$\sigma_{mk}^n(t) = \frac{\tilde{E}_n}{1+\nu_n}\left[\frac{\nu_n}{1-2\nu_n}\Theta^n(t)\delta_{mk} + \varepsilon_{mk}^n(t)\right], \qquad (2)$$

where $\tilde{E}_n$ is the Voltaire operator having the following form:

$$\tilde{E}_n\varphi(t) = E_{0n}\left[\varphi(t) - \int_0^t R_{En}(t-\tau)\varphi(\tau)d\tau\right] \qquad (3)$$

here E_{0n} - instantaneous modulus of elasticity, R_{En} – is the core of relaxation.

Given (1), the function of time in equality (3) will be $\varphi(t) = \exp(-i\omega t)$ with a slowly changing amplitude. Assuming the smallness of the integral term $\int_0^\infty R(\tau)d\tau$, using the freezing method [8], we replace the ratio (3) with an approximate:

$$\tilde{E}_n\varphi(t) \cong E_{0j}\left[1 - \Gamma_n^c(\omega_R) - i\Gamma_n^s(\omega_R)\right]\varphi(t) \equiv \bar{E}_n\varphi(t), \qquad (4)$$

where $\Gamma_E^C(\omega_R) = \int_0^\infty R_E(\tau)\cos\omega_R\tau\, d\tau$, $\Gamma_E^S(\omega_R) = \int_0^\infty R_E(\tau)\sin\omega_R\tau\, d\tau$ - accordingly, the cosine and sine Fourier images of the relaxation kernel of the material, ω_R - the actual value. As an example of a viscoelastic material, we take a three-parameter relaxation core $R_E(t) = Ae^{-\beta_1 t}/t^{1-\alpha_1}$.

For the given boundary conditions, we present the following forms of motion [9]:

$$w(s,\varphi,t) = \sum_{m=1}^\infty\sum_{n=1}^\infty w_{mn}\cos n\varphi \sin\frac{m\pi s}{L},$$
$$\vartheta(s,\varphi,t) = -\sum_{m=1}^\infty\sum_{n=1}^\infty \frac{1}{n} w_{mn}\sin n\varphi \sin\frac{m\pi s}{L}, \qquad (5)$$
$$u(s,\varphi,t) = \frac{\pi r}{L}\sum_{m=1}^\infty\sum_{n=1}^\infty \frac{m}{n^2} w_{mn}\cos n\varphi \cos\frac{m\pi s}{L}.$$

Here u, ϑ and w- displacement of the points of the median surface of the shell in the axial, circumferential and radial directions. Wave numbers m and n characterize the shape of the oscillations: m - the number of half-waves in the axial direction, n- the number of waves in the circumferential direction.

The kinetic energy of the pipe motion is determined by the following equation:

$$K=\frac{1}{2}\rho_T r h_m\int_0^L\int_0^{2\pi}\left(\dot{u}^2+\dot{\vartheta}^2+\dot{w}^2\right)h(\varphi)dsd\varphi=\frac{1}{8}(m_T)\sum_{m-1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{n^2+1}{n^2}+\frac{m^2\pi^2r^2}{n^4L^2}\right)\dot{w}_{mn}^2 \qquad (6)$$

The viscoelastic potential constructed on the basis of the semi-instant theory of multilayer thin shells and approximations (5) has the form:

$$\Pi=\frac{1}{2}r\int_0^L\int_0^{2\pi}\left(B_{1m}\varepsilon_1^2+D_{2m}k_2^2\right)dsd\varphi=$$

$$=\frac{\pi B_{1m}}{4}rL\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[-\left(\frac{m\pi}{L}\right)^2\frac{r}{n^2}w_{mn}+\frac{n-2}{n-1}\frac{w_{mn-1}}{D}+\frac{n+2}{n+1}\frac{w_{mn+1}}{D}\right]^2+ \qquad (7)$$

$$+\frac{\pi D_{2m}L}{4r^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(n^2-1)^2w_{mn}^2 \quad .$$

Using Lagrange equations [22]:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{w}_{mn}}\right)+\frac{\partial \Delta}{\partial \dot{w}_{mn}}-\frac{\partial(K-\Pi-W)}{\partial w_{mn}}=Q_{mn}$$

and dependences (2) - (10), we obtain a connected system of homogeneous differential equations with complex coefficients of the form

$$[A]\{\ddot{w}\}+D_{1m}(1-\Gamma_n^{\circ}(\omega_R))[C]\{w\}=0\,, \qquad (8)$$

Where $\Gamma_n^{\circ}(\omega_R)=\Gamma_n^{C}(\omega_R)+i\Gamma_n^{S}(\omega_R)$, $D_{1m}=\frac{h_m}{1-\nu_{12}\nu_{21}}E_{02}$, E_{02}- the instantaneous modulus of elasticity, $[A]$- is a square diagonal matrix whose elements are determined by recurrent formulas: $a_{nn}=\frac{n^2+1}{n^2}+\frac{m^2\pi^2r^2}{n^4L^2}$, $[C]$- a square matrix whose elements are defined by the following recurrent formulas :

$$c_{nn}=\frac{n^2+1}{n^2}+\frac{2\beta_m^{\ 2}}{n^4}+\frac{1}{6}\varsigma\lambda^2(n^2-1)(n^2-1+3p_m'),$$

$$c_{n\,n+1}=-2\beta_m\frac{n^2+n+1}{n^2(n+1)^2}, \qquad (9)$$

$$c_{11}=2\beta_m^2\ ,c_{n\,n+2}=-2\beta_m\frac{n^2+2n-3}{2n(n+2)}.$$

Here $\beta_m=\left(\frac{m\pi}{L}\right)^2rR$ and $p_m'=\frac{p_mr^3}{3D_{2m}}$- dimensionless parameters, $D_{2m}=\frac{h_m^3}{1-\nu_{12}\nu_{21}}E_{02}$.

Analysis of the structure of the matrix [C] shows that the generalized coordinates are related to each other. The interaction of generalized coordinates is caused by elastic bonds, the intensity of which is characterized by non-diagonal elements of the matrix [C] and depends on the length of the pipe $L=\theta_0R$, where θ_0 - central corner. The shorter the pipe, the greater the number of half-waves *m* on the segment *L* and the greater the curvature parameter *r/R*, the stronger their interaction.

With an increase in the radius of curvature R, the interaction of generalized w_{mn} coordinates weakens

Evaluation of the flexibility of curved viscoelastic composite pipes

Depending on the boundary conditions and geometric factors, we investigate the flexibility of samples of viscoelastic composite pipes. The geometric characteristics of the samples are given in Table 1.

Table 1. Geometric characteristics of samples

Number	*R*, mm	r, mm	*h*, mm	φ_0	*r/R*	*h/r*	λ
1	835	80	4,2	180°	1/10	1/20	0,50
2	1250	80	4,2	180°	1/15	1/20	0,70

The samples are made of organoplasty Kevlar 49/PR-286 with physical characteristics $E_\alpha = 64{,}10GPa,\ E_\beta = 5{,}38GPa,\ G_{\alpha\beta} = 2{,}07GPa, \nu_{\alpha\beta} = 0{,}35$ and parameters of the relaxation core $A = 0{,}048;\ \beta_1 = 0{,}05;\ \alpha_1 = 0{,}10$. The number of layers is six. Effective elastic constants with a wall, as a multilayer orthotropic body, depending on the reinforcement, are given in [10].

Analysis of complex natural frequencies of oscillations

Let us consider the spectra of the lowest complex frequencies and the corresponding eigenforms of articulated multilayer curved viscoelastic pipes with parameters: r=80mm, h/r =1/40, parameters of the relaxation core - $A = 0{,}048;\ \beta_1 = 0{,}05;\ \alpha_1 = 0{,}10$ and reinforcement angle at $\varphi_m = \pm 80^0$ depending on the initial curvature *r/R*=1/40, 1/20, 1/15. The pipes have the same length L =2,5m, but different bending angles - θ_0 =45°, 90°, 135°, 180°. Material - organoplasties Kevlar 49/PR-286. The number of layers is six. The solution of homogeneous differential equations (8) is sought in the form

$$\{w\} = \{w_{mn}\} e^{-i\omega t}, \qquad (10)$$

where w_{mn} - amplitudes of generalized displaced, $\omega = \omega_R + i\omega_I$ - complex frequency.

Substituting (10) into (8), we obtain the following homogeneous algebraic equations

$$(-\omega^2 [A] + D_{1m}(1 - \Gamma_n^\circ(\omega_R))[C])\{w_{mn}\} = 0.$$

The frequency equation of the eigenvalue problem is written as follows:

$$[-\omega^2 [A] + D_{1m}(1 - \Gamma_n^\circ(\omega_R))[C]] = 0 \quad . \qquad (11)$$

The roots of the characteristic equation (11) are found by the Muller method.

Table 2. Change in the real part of the complex natural frequency depending on r/R

r/R	Real parts are complex natural frequencies ω_{Rm1}, Hz			
	m=1	m=2	m=3	m=4
1/10	-	62,3	153,5	515,1
1/15	-	83,7	170,2	486,6

1/20	-	94,4	190,9	455,2
1/40	-	104,6	216,0	433,1
Straight pipe	27,6	110,3	248,3	441,4

Table 2 shows the change in the real part of the complex frequency from the initial curvature r/R and bending angle θ_0 accordingly. It can be seen from Table 2 that with a decrease in the bending angle θ_0 real parts of the lowest complex frequency ($\omega_{Rm\,1}$), relevant n=1 and m=2,3,4 forms, increase. And the real parts are higher frequencies $\omega_{Rm\,2}$ and $\omega_{Rm\,3}$, with appropriate n=2,3 and m=1,2,3,4 forms, on the contrary, decrease. And in the limit – they approach the natural oscillation frequencies of a straight composite elastic pipe.

Conclusion

1. Based on the approximate energy method, a resolving system of equations of free oscillations of thin-walled curved composite viscoelastic pipes with hinged supports is constructed. 2. The layered-fibrous STRUCTURE and anisotropy of the material are taken into account. In a particular case, the resolving equations describe the free oscillations of a cylindrical shell and a hinged straight-rod. The resulting equations correspond to known classical solutions.

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